AIChE

Dynamic Operability Analysis of Nonlinear Process Networks Based on Dissipativity

Osvaldo J. Rojas, Ridwan Setiawan, and Jie Bao
School of Chemical Sciences and Engineering, The University of New South Wales,
Sydney NSW 2052, Australia

Peter L. Lee
The University of South Australia, The Chancellery, Adelaide SA 5000, Australia

DOI 10.1002/aic.11723
Published online March 11, 2009 in Wiley InterScience (www.interscience.wiley.com).

Most modern chemical plants are complex networks of multiple interconnected nonlinear process units, often with multiple recycle and by-pass streams and energy integration. Interactions between process units often lead to plant-wide operability problems (i.e., difficulties in process control). Plant-wide operability analysis is often difficult due to the complexity and nonlinearity of the processes. This article provides a new framework of dynamic operability analysis for plant-wide processes, based on the dissipativity of each process unit and the topology of the process network. Based on the concept of dissipative systems, this approach can deal with nonlinear processes directly. Developed from a network perspective, the proposed framework is also inherently scalable and thus can be applied to large process networks. © 2009 American Institute of Chemical Engineers *AIChE J,* 55: 963–982, 2009
Keywords: plant-wide control, nonlinear systems, process networks, process operability

Introduction

Process control has been playing an increasingly important role in the process industries as more process integration and tighter operating conditions are putting greater demands on control system performance. The traditional approach to process design and control has been to perform the design of the process and the control system sequentially.[1] As little consideration is given to dynamic operability in the process design stage, the outcome of this approach sometimes is a plant whose dynamic characteristics lead to serious operating difficulties associated with significant economic penalties.[2] It is well known that a process design fundamentally determines the inherent operability properties of the process since it imposes limitations on control performance regardless of what control method is implemented. Therefore, process operability analysis should be performed in the process design stage for better integration of process design and control.[1]

Modern chemical plants have two important features: (1) they are networks of significant number of process units with recycle and heat integration configurations; (2) process units are generally nonlinear. Interactions between the process units are often the main causes of operability problems. As such, it is important to develop appropriate tools to quantitatively assess the effects of nonlinear dynamics of process units and the interactions between them on plant-wide operability.[3] In the past two decades, a number of operability analysis tools have been reported. Many of these were developed for linear processes, based on, e.g., process zero dynamics, minimum singular values and loop interactions.[4–6] For nonlinear processes, a direct approach to operability analysis is to solve a dynamic performance optimisation problem numerically. Although this approach allows seamless integration of process and control design,[6–9] the complexity of the resulting problem grows quickly with the size

Correspondence concerning this article should be addressed to J. Bao (j.bao@unsw.edu.au)

of the process plant. Therefore, it is difficult to implement the optimization approach in plant-wide process operability analysis for complex processes.[10] The operability analysis approaches based on open-loop process models are much more appealing due to their relative simplicity compared with the optimization approach. For example, Vinson and Georgakis proposed a steady-state operability analysis based on input-output spaces.[11] Hernjak et al.[12] proposed a qualitative process operability analysis method based on three main process characteristics: the degree of nonlinearity, process dynamics and degree of interaction between loops. Research effort has also concentrated on the nonlinear phenomena that is known to generate operability difficulties such as multiple equilibrium points, limit cycles, chaotic behavior and bifurcations, e.g.,[13–17] However, none of the above open-loop methods addresses the plant-wide operability analysis problem from a network perspective.

In this article, we propose a framework for plant-wide operability analysis of nonlinear process networks, that is to address plant-wide open-loop stability, stabilizability and the achievable dynamic control performance in terms of disturbance attenuation. This approach uses dissipative systems theory as the key enabling tool, which has been shown useful in stability analysis for process networks.[18] The input-output properties of each subsystem can be represented by its dissipativity. Our recent studies have shown that the dissipativity of a nonlinear process is related to its dynamic operability.[19–21] The topology of the process network can be modeled efficiently with a suitable interaction matrix. From the dissipativity of each process unit and the network topology, the dissipativity of the plant-wide process can then be easily determined, which in-turn, indicates the operability of the process network. This approach has several advantages. First, it reduces the modeling costs associated with the analysis of large and complex process networks since the approach only requires the individual models of smaller and simpler process units. Second, the approach is inherently scalable since the addition of any number of process units to the network requires only simple recalculation. Third, the approach is able to quantify how the disturbance to one unit propagates throughout the entire network influencing other variables of interest.

The proposed approach can also be used to determine suitable control structures for plant-wide control, which has been deemed a key issue related to plant-wide control of complex processing plants.[3,22–24] The answer is closely related to the specification of explicit and implicit production and operational objectives for the processing plant.[23] A formal statement of the problem requires the formulation of a multi-objective optimization problem which, in turn, is very difficult to solve owing to its inherent complexity.[23] In practice, plant-wide control is addressed by adopting a hierarchical design procedure so that the design problem can be decomposed into several stages that are simpler to analyze and solve.[3] The framework described in this article is useful for assessing the operability properties of the network for a given selection of controlled and manipulated variables.

A distinctive feature of the plant-wide control framework proposed in the current work is that it explicitly distinguishes the physical interconnecting flows between process units (such as mass and energy flows) from the information or control flows (such as measurements and actuation signals used to regulate the process units and the entire network). This allows one to clearly identify possible sources of stability and/or operability difficulties.

This article is organized as follows. First, a new approach to represent plant-wide processes is presented. The concept of dissipative systems is introduced in the following section. Next, the dissipativity-based plant-wide operability analysis is elaborated, including the features and relevant issues of the proposed method. A case study of a heat exchanger network is then presented to illustrate the dissipativity-based network analysis approach, followed by the conclusion.

A Network Perspective of Plant-Wide Analysis

Modern chemical processing plants are complex networks of multiple interconnected processing units. The fundamental interconnecting flows between units are physical mass and energy flows. Superimposed to this network of physical flows is the network of information flows that connects the variety of sensors that monitor the plant to data acquisition systems and local/supervisory controllers. From there the information flows connect back to the series of actuators that regulate the plant. The dynamic behavior of physical processes is governed by physical laws. For chemical processes, the dynamics of the physical flows are dictated by the conservation of mass and energy. On the other hand, the behavior of information flows is totally independent from physical laws.[25] It is therefore important to make a distinction between the physical and information flows.

To treat the interconnecting and information flows explicitly in this analysis, it is assumed that the i-th process unit, denoted as Σ_i, pertaining to the network has a set of physical interconnecting inputs $\tilde{u}_i$ and a set of manipulated inputs $\hat{u}_i$ as shown in Figure 1. In this article, all input and output variables are column vectors. The term d_i indicates any disturbances acting on the process unit. The output vector y_i in Figure 1 encompasses both the physical interconnecting outputs of the unit as well as any sensor measurement outputs available for control. The physical interconnecting outputs $\tilde{y}_i$ of the process unit are a sub-set of the output vector y_i given by:

$$\tilde{y}_i = F_I y_i \tag{1}$$

where F_I is a constant selector matrix. For example, if all the components of the output vector y_i are physical interconnecting outputs then F_I is the identity matrix. Similarly, the measured outputs $\hat{y}_i$ of the process unit are a sub-set of the output vector y_i given by:

$$\hat{y}_i = F_C y_i \tag{2}$$

where F_C is also a constant selector matrix. Observe that depending on the specific set of sensors available, an element of the output vector y_i could be an interconnecting output and/or a measured output.

Consider a process network formed by N process units with the general structure shown in Figure 1. Combining similar inputs and outputs from each process unit into single composite vectors as follows,

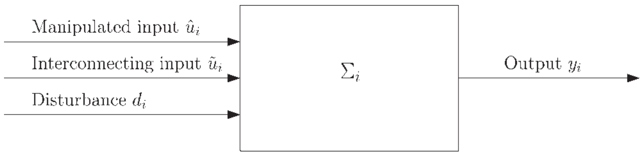

Figure 1. The *i*-th process unit: inputs and outputs.

$$\begin{aligned}\hat{u}^T &= [\hat{u}_1^T \quad \hat{u}_2^T \quad \dots \quad \hat{u}_N^T] \\ \tilde{u}^T &= [\tilde{u}_1^T \quad \tilde{u}_2^T \quad \dots \quad \tilde{u}_N^T] \\ d^T &= [d_1^T \quad d_2^T \quad \dots \quad d_N^T] \\ y^T &= [y_1^T \quad y_2^T \quad \dots \quad y_N^T] \\ \hat{y}^T &= [\hat{y}_1^T \quad \hat{y}_2^T \quad \dots \quad \hat{y}_N^T] \\ \tilde{y}^T &= [\tilde{y}_1^T \quad \tilde{y}_2^T \quad \dots \quad \tilde{y}_N^T]\end{aligned} \tag{3}$$

it is possible to describe the entire process network using the diagram shown in Figure 2. The physical interconnecting flows among process units in the network are modeled using a static interconnecting matrix H so that:

$$\begin{aligned}\tilde{u} &= -H\tilde{y} \\ &= -HF_1 y \\ &= -\tilde{H}y\end{aligned} \tag{4}$$

Observe that the physical interconnecting flows use negative sign convention.

The information flows carried by the vector of measured outputs $\hat{y}$ are processed by the control system and actions are taken through the manipulated variables of each process unit. At this point no specific assumption is required regarding the structure of the plant-wide control system shown in Figure 2.

Example 1. *The aforementioned network representation approach can be illustrated using a simple example. Consider a process consisting of two tanks with the same cross sectional area A and same outlet valve coefficient C_v connected in series with a recycle stream flowing from the second tank to the first tank at constant recycle ratio (R_r) (as shown in Figure 3). It is assumed that all the streams have the same liquid properties. Thus, the first tank can be modeled as follows:*

$$\begin{aligned}\frac{dh_1(t)}{dt} &= -\frac{C_v}{A}\sqrt{h_1(t)} + \frac{1}{A}(F_0(t) + F_1(t) + F_R(t)) \\ y(t) &= \begin{bmatrix} C_v\sqrt{h_1(t)} \\ h_1 \end{bmatrix}\end{aligned} \tag{5}$$

And the second tank can be modeled as follows:

$$\begin{aligned}\frac{dh_2(t)}{dt} &= -\frac{C_v}{A}\sqrt{h_2(t)} + \frac{1}{A}F_2(t) \\ y(t) &= C_v\sqrt{h_2(t)}\end{aligned} \tag{6}$$

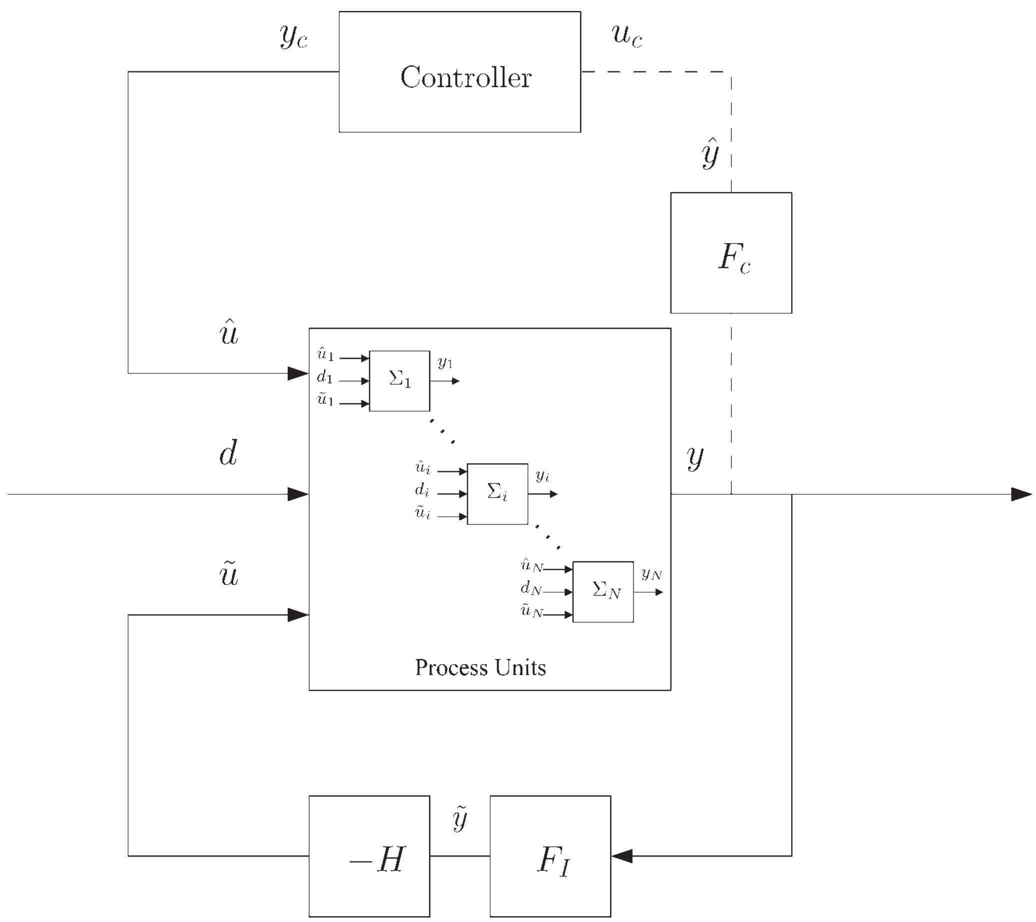

Figure 2. Process network with interconnecting flow (continuous line) and information flow (dashed line).

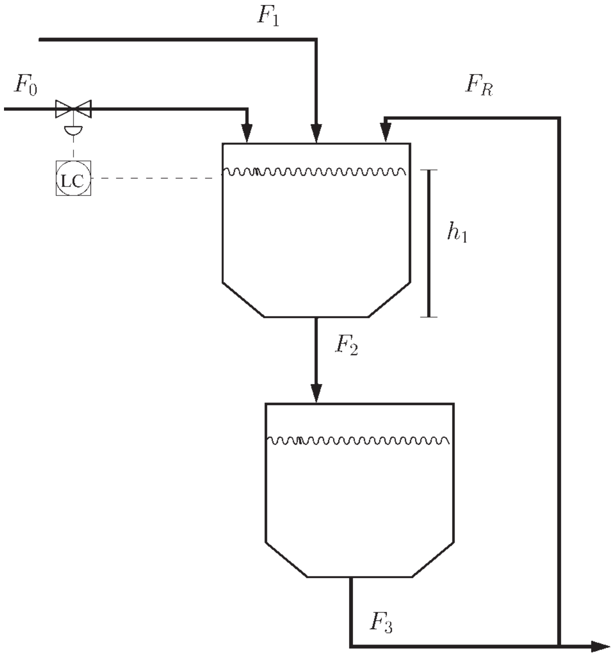

Figure 3. A double tank system.

This process configuration can then be represented using the network perspective described as follows:

(1) Each tank is treated as a subsystem and thus the process system is decomposed into smaller subsystems (Σ_i, $i = 1, 2$).

(2) The inputs of each subsystem are represented as $[\hat{u}_i^T \quad \tilde{u}_i^T \quad d_i^T]^T$ $(i = 1, 2)$, distinguishing the manipulated variables, interconnecting flows and the disturbances. The output is denoted as y_i $(i = 1, 2)$. Subsystem Σ_1 (Tank 1) has $F_0(t)$ as the manipulated input. $F_1(t)$ is neither manipulated nor connected to another subsystem and thus is regarded as the disturbance. $F_R(t)$ is the interconnecting input as it is connected to another subsystem. The outputs y_1 from Σ_1 include the outlet flow to subsystem Σ_2 (Tank 2) $F_2(t)$ and the liquid level of the first tank $h_1(t)$. Similarly, subsystem Σ_2 has $F_2(t)$ as the interconnecting input and $F_3(t)$ as the output:

$$\begin{aligned} &\hat{u}_1 = F_0, \quad \tilde{u}_2 = F_2 \\ &\tilde{u}_1 = F_R, \quad y_2 = F_3 \\ &d_1 = F_1 \\ &y_1 = [F_2 \quad h_1]^T \end{aligned} \tag{7}$$

(3) The network perspective as shown in Figure 2 is then applied to the entire plant-wide system. All the inputs to both subsystems are grouped in a column vector $[\hat{u}^T \quad \tilde{u}^T \quad d^T]^T$ and all the outputs are grouped in a column vector y.

$$\begin{aligned} &\hat{u} = F_0 \\ &\tilde{u} = [F_R \quad F_2]^T \\ &d = F_1 \\ &y = [F_2 \quad h_1 \quad F_3]^T \end{aligned} \tag{8}$$

(4) Constant selector matrices F_1 and F_c are chosen to represent the physical flows and information flows:

$$\begin{aligned} \tilde{y} = F_1 y &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & R_r \end{bmatrix} \begin{bmatrix} F_2 \\ h_1 \\ F_3 \end{bmatrix} \\ \hat{y} = F_C y &= \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} F_2 \\ h_1 \\ F_3 \end{bmatrix} \end{aligned} \tag{9}$$

(5) The interconnecting matrix H is formulated as follows:

$$\tilde{u} = -HF_1 y = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & R_r \end{bmatrix} \begin{bmatrix} F_2 \\ h_1 \\ F_3 \end{bmatrix} \tag{10}$$

It is worth pointing out that the proposed network representation is completely general, applicable to both linear and nonlinear processes, regardless of their complexity. It can be used to represent processes in the form of discrete-time, continuous or hybrid systems as well as distributed parameter systems (represented with partial differential equations).

Dissipative Systems Theory for Process Network Analysis

Dissipative systems

The plant-wide operability analysis framework described in the current article adopts a dissipative systems theory approach. The concept of dissipative systems was originally introduced by Willems[26,27] and is an extension of the concept of passivity. Passivity and dissipativity have been recognized as effective tools for nonlinear system analysis and control.[28,29] Dissipative systems are those for which the increase in stored energy is bounded by the amount of energy supplied by the environment, i.e., dissipative systems can only dissipate but not generate energy. As dissipativity represents the input-output properties of nonlinear processes, the dissipative systems theory is well suited for the analysis of the effects of interactions in large scale process networks. In what follows, the key concepts related to dissipative systems theory and their relevance to the analysis of nonlinear process networks are reviewed. For ease of exposition, detailed definitions are omitted but can be found in the Appendix.

The concept of dissipativity relates to the input-output property of dynamical systems.[30] Consider a dynamical system Σ defined through the input space U, output space Y, state space X and two functions: the state transition function $\psi(t, t_0, x_0, u)$ which maps $\mathbf{R}_2^+ \times X \times U$ into X, and the read-out function $r(\psi(t, t_0, x_0, u), u(t))$ which maps $X \times U$ into Y, where t is the time variable, t_0 represents the initial time, $x_0 \in X$ represents the state value at t_0, $u \in U$ is the input trajectory as a function of time, and $\mathrm{R}_2^+ = \{(t_2, t_1) | t_2 \geq t_1$ and $t_1, t_2 \in \mathrm{R}\}$ (see Definition A.1 for details).

For $u(t) \in U$ and $y(t) \in Y$, define a real valued function $w(u(t), y(t)) : U \times Y \to \mathrm{R}$, called the supply rate (see Definition A.2 for details). The dynamic behavior of a dissipative system is constrained by a so-called dissipation inequality given by[26]:

$$\phi(x(\tau)) - \phi(x(0)) \leq \int_0^{\tau} w(u(t), y(t))dt, \quad \forall\ \tau \geq 0 \quad (11)$$

where $x(t) \in X$ with $x(0)$ as the initial state at $t = 0$ and $\phi(x) : X \rightarrow \mathrm{R}^{+}$ is a nonnegative function of the states (x) called the storage function (see Definition A.3). The supply rate $w(u, y)$ can then be understood as the rate of energy transferred to the system from the environment whilst the storage function $\phi(x)$ can be understood as the total internal energy contained in the system. The dissipation inequality (11) formalizes the property that the increase in stored energy is never greater than the integral of the supply rate (energy supplied by the environment).

In general, functions $\phi(x)$ and $w(u, y)$ can be any generalized energy function (similar to a Lyapunov function) and any abstract power function respectively. However, storage functions and supply rates with physical meanings can be adopted so that inequality (11) represents the dissipation of physical energy. Ydstie and co-workers revealed the link between thermodynamics and passivity.[30–33] By using entropy based storage functions, the passivity of a class of chemical processes (called process systems) can be established.[34] In addition, Hangos et al.[35] also combined Hamiltonian systems representation with thermodynamic related dissipativity and observed the physical meanings of the supply rates and storage energy functions. Using the aforementioned approaches, process dissipativity and passivity can be determined without the need of a detailed process model, relying instead on physical insights or knowledge.

Dissipativity can represent most of the important input-output properties by adopting different supply rates.[29] A process is said to be passive, input feedforward passive and output feedback passive if it is dissipative with respect to the following supply rates respectively (it is assumed that the process has the same number of inputs and outputs, i.e. $m = p$):

- Passivity

$$w(u(t), y(t)) = y(t)^T u(t) \quad (12)$$

- Input feedforward passivity (IFP):

$$w(u(t), y(t)) = y(t)^T u(t) - \nu u(t)^T u(t), \quad \nu > 0 \quad (13)$$

- Output feedback passivity (OFP):

$$w(u(t), y(t)) = y(t)^T u(t) - \rho y(t)^T y(t), \quad \rho > 0 \quad (14)$$

As (12) is the inner product of the input and output, passivity relates to the phase condition of a system. A nonlinear input feedforward passive system is minimum phase (i.e., with stable zero dynamics).[29] An output feedback passive system has bounded gain (input-output stable).[29] Furthermore, if a process is dissipative with the following supply rate:

$$w(u(t), y(t)) = y(t)^T y(t) - \gamma^2 u(t)^T u(t), \quad \gamma > 0 \quad (15)$$

then its L_2 gain is bounded by γ.

If a process is dissipative with respect to the supply rate given in (13) or (14) with $\nu < 0$ or $\rho < 0$, then the process is in shortage of IFP or OFP. In a feedback system, the shortage of IFP (or OFP) of one subsystem can be compensated by the excess of OFP (or IFP) of another subsystem to maintain closed-loop input-output stability (or Lyapunov stability if an additional zero state detectability condition is satisfied).[29]

In this article, we consider nonlinear processes which are dissipative with respect to a more general quadratic supply rate,[36] i.e.:

$$w(u(t), y(t)) = y(t)^T Q y(t) + 2y(t)^T S u(t) + u(t)^T R u(t) \quad (16)$$

where $Q \in \mathrm{R}^{p\times p}$, $S \in \mathrm{R}^{p\times m}$, $R \in \mathrm{R}^{m\times m}$ are constant matrices with Q and R symmetric (in this case, the assumption $m = p$ is not required). This type of nonlinear processes is said to be (Q, S, R)-dissipative to emphasize the special structure of the associated supply rate. The quadratic supply rate is the combination of supply rates for IFP and OFP. Therefore, (Q, S, R)-dissipativity carries the information on the (nonlinear counterpart of the) phase (given by the cross term $y^T Su$), the process gain (given by the quadratic term $y^T Qy + u^T Ru$) and input-output stability of the process. As such, (Q, S, R)-dissipativity implies dynamic operability of nonlinear processes, as shown in our recent work.[21]

The following example shows how the (Q, S, R)-dissipativity of a nonlinear system can be determined using Eq. 16 directly.

Example 2. *Consider the second tank in Figure 3. The system's model was represented in Eq. 6, where $u(t) = F_2(t)$. In this example, the momentum of the liquid in the tank is used as the physical storage function.*

$$\begin{aligned} \phi(h_2) &= mv \\ \phi(h_2) &= \rho C_v h_2^{\frac{3}{2}} \end{aligned} \quad (17)$$

where m, v, ρ, C_v, h_2 correspond to the mass of liquid in the tank, the rate of change of the liquid level, the density of the liquid, the outlet valve coefficient and the liquid level respectively. The time derivative of $\phi(h_2)$ is then:

$$\begin{aligned} \dot{\phi}(h_2) &= \frac{3}{2}\rho C_v \sqrt{h_2}\frac{dh_2}{dt} \\ &= \frac{3}{2}\rho C_v \sqrt{h_2}\left(-\frac{C_v}{A}\sqrt{h_2} + \frac{1}{A}F_2\right) \\ &= -\frac{3}{2}\rho\frac{C_v^2}{A}h_2 + \frac{3}{2}\frac{\rho C_v}{A}\sqrt{h_2}F_2 \\ &\leq -\frac{\rho}{A}C_v^2 h_2 + 2(C_v\sqrt{h_2})\frac{3\rho}{4A}F_2 \\ &\leq y^T Q y + 2y^T S u \end{aligned} \quad (18)$$

where:

$$\begin{aligned} y &= C_v\sqrt{h_2} \\ u &= F_2 \end{aligned} \quad (19)$$

and

$$\begin{aligned} Q &= -\frac{\rho}{A} \\ S &= \frac{3\rho}{4A} \end{aligned} \quad (20)$$

From Inequality (18), it can be concluded that the tank system is (Q, S, R)-dissipative with Q and S given in (20) and R = 0.

More specifically, for control-affine nonlinear systems given by:

$$P : \begin{cases} \dot{x} = f(x) + G(x)u \\ y = h(x) + J(x)u \end{cases} \tag{21}$$

(where the columns of $G(x)$ and $J(x)$ are smooth vector fields), there is a more systematic way to establish its (Q, S, R)-dissipativity, as described in the following theorem:

Theorem 1 *((Q, S, R)-dissipativeness).*[36] *A necessary and sufficient condition for the system P in (21) to be (Q, S, R)-dissipative is that there exist real functions $\phi(x) : \mathrm{R}^n \rightarrow \mathrm{R}^+$, $l(x) : \mathrm{R}^n \rightarrow \mathrm{R}^q$, and $W(x) : \mathrm{R}^n \rightarrow \mathrm{R}^{q\times m}$ (for some integer q) satisfying:*

$$\begin{aligned}
&(i) \quad \phi(x) \geq 0, \ \phi(0) = 0 \\
&(ii) \quad \frac{\partial \phi^T}{\partial x} f(x) = h(x)^T Q h(x) - l(x)^T l(x) \\
&(iii) \quad \frac{1}{2} G(x)^T \frac{\partial \phi}{\partial x} = (QJ(x) + S)^T h(x) - W(x)^T l(x) \\
&(iv) \quad R + J(x)^T S + S^T J(x) + J(x)^T Q J(x) = W(x)^T W(x)
\end{aligned} \tag{22}$$

Consider a more general dynamical system whose state-space representation is defined as follows:

$$\Sigma : \begin{cases} \dfrac{dx(t)}{dt} = f(x(t), u(t)) \\ y(t) = g(x(t), u(t)) \end{cases} \tag{23}$$

where $x \in X \subset \mathrm{R}^n$ is the process state vector, $u \in U \subset \mathrm{R}^m$ is the input vector and $y \in Y \subset \mathrm{R}^p$ is the output vector. It is assumed that $f(x,u)$ and $g(x,u)$ are smooth vector fields, and that, without loss of generality, Σ has an equilibrium point at u = 0, x = 0 and y = 0. Thus $f(0,0) = 0$ and $h(0,0) = 0$. If the system Σ is (Q, S, R)-dissipative, then its input-output stability can be inferred directly from the properties of the matrix Q.

Theorem 2 *(Stability of (Q, S, R)-dissipative systems).*[37] *Consider a nonlinear system Σ (as given in (23)) which is (Q, S, R)-dissipative, with $x_0 = 0$. If $Q < 0$, the system is L_2 stable, i.e., it has finite L_2 gain:*

$$\|y_\tau\|_{L_2} \leq \gamma \|u_\tau\|_{L_2} \tag{24}$$

where the subscript τ denotes truncation, e.g.:

$$y_\tau(t) \triangleq \begin{cases} y(t), & 0 \leq t \leq \tau \\ 0, & t > \tau \end{cases} \tag{25}$$

Furthermore, if Σ is zero-state detectable, (i.e., if $u(t) = 0$ and $y(t) = 0$ then $\lim_{l\to\infty} x(t) = 0$), then the free system:

$$\dot{x} = f(x, 0) \tag{26}$$

obtained from (23) with $u(t) = 0$, has an asymptotically stable equilibrium x = 0 (in the sense of Lyapunov stability).

The condition $Q < 0$ can be interpreted as a form of OFP, which implies finite L_2 gain.[38] To formalize this, it is assumed that the process input $u(t)$ belongs to the space of piecewise continuous, square integrable functions (called extended L_2 space), i.e.

$$\int_0^\tau u(t)^T u(t)t \ < \ \infty \tag{27}$$

for any $0 \leq \tau < \infty$.

Then we have:

Theorem 3 *(L_2 gain of (Q, S, R)-dissipative systems).*[37] *Consider a nonlinear (Q, S, R)-dissipative process Σ with $Q < 0$. The L_2 gain γ of Σ is bounded by:*

$$\gamma \leq \|\hat{Q}^{1/2}\| (\alpha + \|\hat{Q}^{1/2} S\|) \tag{28}$$

where $\|.\|$ is the (induced) matrix 2-norm. That is, $\|A\| = \bar{\sigma}(A)$, $\hat{Q} = -Q$ and $\alpha > 0$ is a finite scalar such that:

$$R + S^T \hat{Q}^{-1} S - \alpha^2 I \leq 0 \tag{29}$$

Network of dissipative systems

To apply dissipative systems theory to the analysis of a network of nonlinear processes, the following key assumption is made: each process unit in the network is (Q_i, S_i, R_i)-dissipative with $\begin{bmatrix} \hat{u}_i^T & \tilde{u}_i^T & d_i^T \end{bmatrix}^T$ as the input vector and y_i as the output vector as shown in Figure 1. That is:

$$\frac{d\phi_i(x)}{dt} \leq y_i^T Q y_i + 2y_i^T S \begin{bmatrix} \hat{u}_i \\ \tilde{u}_i \\ d_i \end{bmatrix} + \begin{bmatrix} \hat{u}_i^T & \tilde{u}_i^T & d_i^T \end{bmatrix} R \begin{bmatrix} \hat{u}_i \\ \tilde{u}_i \\ d_i \end{bmatrix} \quad \forall i \tag{30}$$

where S_i and R_i can be represented as:

$$\begin{aligned} S_i &= \begin{bmatrix} S_{1i} & S_{2i} & S_{3i} \end{bmatrix} \\ R_i &= \begin{bmatrix} R_{1i} & R_{4i} & R_{5i} \\ R_{4i}^T & R_{2i} & R_{6i} \\ R_{5i}^T & R_{6i}^T & R_{3i} \end{bmatrix} \end{aligned} \tag{31}$$

using sub-matrices. One simple way to analyze the dissipative properties of the entire nonlinear process network is to consider a storage function for the network given by:

$$\phi(x) = \sum_{i=1}^N \phi_i(x_i) \tag{32}$$

Using the aforementioned storage function and the dissipation inequality (30) for each unit in the network, it is straightforward to show that the process network satisfies the dissipation inequality:

$$\frac{d\phi(x)}{dt} \leq y^T \mathbf{Q} y + 2y^T \mathbf{S} \begin{bmatrix} \hat{u} \\ \tilde{u} \\ d \end{bmatrix} + [\hat{u} \quad \tilde{u} \quad d] \mathbf{R} \begin{bmatrix} \hat{u} \\ \tilde{u} \\ d \end{bmatrix} \tag{33}$$

where

$$\mathbf{S} = [\,\mathbf{S}_1 \quad \mathbf{S}_2 \quad \mathbf{S}_3\,]$$
$$\mathbf{R} = \begin{bmatrix} \mathbf{R}_1 & \mathbf{R}_4 & \mathbf{R}_5 \\ \mathbf{R}_4^{\mathbf{T}} & \mathbf{R}_2 & \mathbf{R}_6 \\ \mathbf{R}_5^{\mathbf{T}} & \mathbf{R}_6^{\mathbf{T}} & \mathbf{R}_3 \end{bmatrix} \tag{34}$$

Matrix $\mathbf{R}$ is symmetric, consisting of six independent submatrices. The constant matrices $\mathbf{Q}$, $\mathbf{S}_j (j = 1, \ldots, 3)$ and $\mathbf{R}_j (j = 1, \ldots, 6)$ can be derived directly from the supply rates of individual process units in the network, as shown below:

$$\begin{aligned} \mathbf{Q} &= \mathrm{diag}\{Q_i\}, \quad i = 1, \ldots, N \\ \mathbf{S} &= \mathrm{diag}\{S_{ji}\}, \quad j = 1, \ldots, 3, \;\; i = 1, \ldots, N \\ \mathbf{R} &= \mathrm{diag}\{R_{ji}\}, \quad j = 1, \ldots, 6, \;\; i = 1, \ldots, N \end{aligned} \tag{35}$$

where (Q_i, S_i, R_i) describe the supply rate associated with i-th process unit in the network.

Plant-Wide Operability Analysis

In this article, we study the following key aspects of dynamic process operability: (1) plant-wide open-loop stability, (2) stabilizability and (3) achievable dynamic control performance in terms of disturbance attenuation.

Open-loop stability of process networks

In many cases, it is useful to ascertain whether a process network is stable in open-loop, i.e., without the influence of the control system. This information can be used to suggest changes to the process network flowsheet to ameliorate some instability effects introduced by large recycle streams or heat integration schemes. Dissipative systems theory can provide this information in a direct and simple way without the need of complex calculations. The key tool for stability analysis of dissipative systems is Theorem 2. To analyze the stability properties of the process network in open-loop one needs to quantify the effect of the physical interconnecting flow on the dissipation inequality derived in (33).

Proposition 1 *(Dissipativity of process network in open-loop). Consider a nonlinear process network as shown in Figure 2 with a set of physical interconnecting flows given by:*

$$\tilde{u} = -\tilde{H}y \tag{36}$$

and no plant-wide control system in place. Assume that the collection of units that form the process network satisfies the dissipation inequality in (33), i.e.,

$$\frac{d\phi(x)}{dt} \le y^T\mathbf{Q}y + 2y^T[\,\mathbf{S}_1 \quad \mathbf{S}_2 \quad \mathbf{S}_3\,]\begin{bmatrix} \hat{u} \\ \tilde{u} \\ d \end{bmatrix} + [\,\hat{u} \quad \tilde{u} \quad d\,]\begin{bmatrix} \mathbf{R}_1 & \mathbf{R}_4 & \mathbf{R}_5 \\ \mathbf{R}_4^{\mathbf{T}} & \mathbf{R}_2 & \mathbf{R}_6 \\ \mathbf{R}_5^{\mathbf{T}} & \mathbf{R}_6^{\mathbf{T}} & \mathbf{R}_3 \end{bmatrix}\begin{bmatrix} \hat{u} \\ \tilde{u} \\ d \end{bmatrix} \tag{37}$$

Then the process network with the physical interconnecting flow (36) is (Q, S, R)-dissipative and satisfies:

$$\begin{aligned} \frac{d\phi(x)}{dt} \le\; & y^T(\mathbf{Q} - \mathbf{S}_2\tilde{H} - \tilde{H}^T\mathbf{S}_2^T + \tilde{H}^T\mathbf{R}_2\tilde{H})y \\ & + 2y^T[\,\mathbf{S}_1 - \tilde{H}^T\mathbf{R}_4^T \quad \mathbf{S}_3 - \tilde{H}^T\mathbf{R}_6\,]\begin{bmatrix} \hat{u} \\ d \end{bmatrix} \\ & + [\,\hat{u}^T \quad d^T\,]\begin{bmatrix} \mathbf{R}_1 & \mathbf{R}_5 \\ \mathbf{R}_5^T & \mathbf{R}_3 \end{bmatrix}\begin{bmatrix} \hat{u} \\ d \end{bmatrix} \end{aligned} \tag{38}$$

Proof. The proposition can be immediately proved by substituting $\tilde{u}$ in (33) with (36). ■

It can be seen that the process network is asymptotically stable if the following condition is satisfied:

$$\mathbf{Q} - \mathbf{S}_2\tilde{H} - \tilde{H}^T\mathbf{S}_2^T + \tilde{H}^T\mathbf{R}_2\tilde{H} < 0 \tag{39}$$

The aforementioned inequality shows that if each process unit is (Q, S, R)-dissipative, then the stability of the process network (whose sub-process units interact via by-pass and recycle streams) without control action depends on the topology of the process network that is described by $\tilde{H}$. Clearly the stability of the process network may not be guaranteed even though each process unit in isolation is stable, i.e., even though Q_i are all negative definite for $i = 1, \ldots, N$. To our knowledge, the aforementioned condition is the first result that explicitly relates the topology of a chemical process network to its plant-wide stability. In the next subsection, we extend the above open-loop (in the sense of feedback control) stability condition to a closed-loop result which can be used to study the stabilizability of process networks based on the dissipativity of process units and the topology of the process network.

Stabilizability of process networks

From a control engineer's point of view, it is often more important to know whether a plant-wide process can be stabilized using feedback control systems. Addressing this issue requires consideration of the influence of the information flow, i.e., the effect of the plant-wide controller. In this work, it is assumed that the controller is (Q_c, S_c, R_c)-dissipative. This assumption may seem, at first, restrictive. However, the authors have shown elsewhere [Theorem 3.2 in (21)] that any stable (possibly nonlinear) input-affine controller is (Q_c, S_c, R_c)-dissipative, provided a certain linear matrix inequality (LMI), relating the matrices Q_c, S_c, R_c, and the controller's direct feed-through matrix, is satisfied. In particular, observe that standard PI-controllers are (Q_c, S_c, R_c)-dissipative (see case study section).

Theorem 4 *(Stabilizability of process networks). Consider the process network shown in Figure 2 and discussed in Proposition 1. Consider a plant-wide controller $\Sigma_c : u_c \rightarrow y_c$ that is (Q_c, S_c, R_c)-dissipative, i.e., one that satisfies the dissipation inequality:*

$$\frac{d\phi_c(x_c)}{dt} \le y_c^T Q_c y_c + 2y_c^T S_c u_c + u_c^T R_c u_c \tag{40}$$

As shown in Figure 2, the controller Σ_c is connected to the process network such that:

$$\begin{aligned} \hat{u} &= y_c \\ u_c &= -\hat{y} = -F_c y \end{aligned} \tag{41}$$

where $\hat{y}$ is the vector of measured variables. Under these conditions, the process network is stabilizable if a set of Q_c, S_c and R_c can be found such that the Q matrix of the closed-loop system with the controller:

$$Q_{cl} = \begin{bmatrix} \mathbf{Q} - \mathbf{S}_2\tilde{H} - \tilde{H}^T\mathbf{S}_2^T + \tilde{H}^T\mathbf{R}_2\tilde{H} + F_c^T R_c F_c & \mathbf{S}_1 - F_c^T S_c^T - \tilde{H}^T\mathbf{R}_4^T \\ \mathbf{S}_1^T - S_c F_c - \mathbf{R}_4\tilde{H} & \mathbf{R}_1 + Q_c \end{bmatrix} t < 0 \tag{42}$$

Proof. Consider the storage function $\hat{\phi} = \phi(x) + \phi_c(x_c)$. Then, clearly, from (38) and (40) we have that:

$$\frac{d\hat{\phi}(\hat{x})}{dt} \leq y^T(\mathbf{Q} - \mathbf{S}_2\tilde{H} - \tilde{H}^T\mathbf{S}_2^T + \tilde{H}^T\mathbf{R}_2\tilde{H})y + 2y^T[\mathbf{S}_1 - \tilde{H}^T\mathbf{R}_4^T \quad \mathbf{S}_3 - \tilde{H}^T\mathbf{R}_6]\begin{bmatrix}\hat{u}\\ d\end{bmatrix} + [\hat{u} \quad d]\begin{bmatrix}\mathbf{R}_1 & \mathbf{R}_5\\ \mathbf{R}_5^T & \mathbf{R}_3\end{bmatrix}\begin{bmatrix}\hat{u}\\ d\end{bmatrix} + y_c^T Q_c y_c + 2y_c^T S_c u_c + u_c^T R_c u_c \tag{43}$$

This leads to:

$$\frac{d\hat{\phi}(\hat{x})}{dt} \leq [y^T \quad y_c^T]\begin{bmatrix} \mathbf{Q} - \mathbf{S}_2\tilde{H} - \tilde{H}^T\mathbf{S}_2^T + \tilde{H}^T\mathbf{R}_2\tilde{H} + F_c^T R_c F_c & \mathbf{S}_1 - F_c^T S_c^T - \tilde{H}^T\mathbf{R}_4^T \\ \mathbf{S}_1^T - S_c F_c - \mathbf{R}_4\tilde{H} & \mathbf{R}_1 + Q_c \end{bmatrix}\begin{bmatrix}y\\ y_c\end{bmatrix} + 2[y^T \quad y_c^T]\begin{bmatrix}\mathbf{S}_3 - \tilde{H}^T\mathbf{R}_6\\ \mathbf{R}_5\end{bmatrix} d + d^T\mathbf{R}_3 d \tag{44}$$

where $\hat{\phi} = \phi(x) + \phi_c(x_c)$ and $\hat{x} = [x^T \quad x_c^T]^T$. Condition (42) immediately follows if the closed-loop system is zero state detectable.

Assume the plant-wide control system $\Sigma_c : u_c \to y_c$ is (Q_c, S_c, R_c)-dissipative and has the following form (with a direct feed-through matrix J):

$$\begin{cases} \dot{x}_c = f_c(x) + G_c(x)u_c \\ y_c = h_c(x) + Ju_c \end{cases} \tag{45}$$

the stabilizability analysis can be performed by solving the following Linear Matrix Inequality (LMI) problem:

Problem 1 (Stabilizability of nonlinear process network). *Given:*

(1) *A nonlinear process network (as shown in Figure 2) satisfying the dissipation inequality (38):*

$$\frac{d\phi(x)}{dt} \leq y^T(\mathbf{Q} - \mathbf{S}_2\tilde{H} - \tilde{H}^T\mathbf{S}_2^T + \tilde{H}^T\mathbf{R}_2\tilde{H})y + 2y^T[\mathbf{S}_1 - \tilde{H}^T\mathbf{R}_4^T \quad \mathbf{S}_3 - \tilde{H}^T\mathbf{R}_6]\begin{bmatrix}\hat{u}\\ d\end{bmatrix} + [\hat{u}^T \quad d^T]\begin{bmatrix}\mathbf{R}_1 & \mathbf{R}_5\\ \mathbf{R}_5^T & \mathbf{R}_3\end{bmatrix}\begin{bmatrix}\hat{u}\\ d\end{bmatrix}$$

(2) *A (Q_c, S_c, R_c)-dissipative plant-wide control system $\Sigma_c : u_c \to y_c$ with a direct feed-through matrix J as shown in (45).*

Find Q_c, S_c, R_c and J such that the LMI given in Equation (42) and

$$R_c + J^T S_c + S_c^T J + J^T Q_c J \geq 0 \tag{46}$$

are satisfied.

The aforementioned problem is convex and can be solved by using any semi-definite programming tools.

The stabilizability condition given in Theorem 4 applies to general nonlinear controllers and thus does not assume controllers to have the form given in (45). The particular controller form is adopted to simplify the problem of determining process stabilizability as it allows the dissipativity of the controller to be specified based on Theorem 1. The additional matrix inequality in (46) is required to guarantee the existence of a (Q_c, S_c, R_c)-dissipative plant-wide control system (see Theorem A.1 in the Appendix).

From Problem 1, it can be seen that the plant-wide stabilizability is affected by both the network topology $\tilde{H}$ that describes the physical interconnections and the choices of controlled variables (described by F_c).

Achievable disturbance attenuation

It is important to quantify the achievable dynamic performance in terms of disturbance attenuation, as part of plant-wide operability analysis. Like open-loop stability and plant-wide stabilizability, achievable disturbance attenuation is an inherent property of the process, independent of the choice of controller. Such operability analysis tools for nonlinear processes are still not generally available. The authors have described an approach based on dissipative systems theory for nonlinear processes.[21] Some of the tools developed in[21] can be adapted for networks of nonlinear processes.

The key element that allows one to develop an analysis tool for plant-wide achievable disturbance attenuation is the close connection between the (Q, S, R)-dissipativity properties of the process network and its finite gain L_2 stability properties. Once this connection is properly established, the achievable control performance in terms of disturbance attenuation for a process network can be assessed by minimizing the L_2 gain that relates the disturbances $d(t)$ affecting the network and the variables $y^*(t)$ considered relevant for dynamic performance. For example, $y^*(t)$ could contain the deviations of the plant product concentrations from their target values. Clearly, a nonlinear process network with a smaller L_2 gain from $d(t)$ to $y^*(t)$ has better dynamic operability characteristics and thus is a preferred design.

It is assumed that the vector $y^*(t)$ of variables relevant to dynamic operability can be defined as follows:

$$y^* = V\hat{y} \tag{47}$$

where V is a constant selector matrix that could also contain the relative weighting assigned to each component of the measurement vector $\hat{y}$ for operability.

Theorem 5 *(Achievable disturbance attenuation). Consider the nonlinear process network shown in Figure 2. Assume that each unit in the network is (Q_i, S_i, R_i)-dissipative. In addition, consider a plant-wide controller $\Sigma_c : u_c \to y_c$ that is (Q_c, S_c, R_c)-dissipative. Suppose that every unit in the network, including the controller Σ_c, are zero state detectable and have zero initial conditions. Let the disturbance vector $d(t)$ acting on the process network be in L_{2e}. Consider the nonlinear process network with input $d(t)$ and output the vector $y^*(t)$ of variables relevant to dynamic operability. If the following linear matrix inequality (LMI):*

$$\begin{bmatrix} \mathbf{R}_3 - \alpha^2 I & \mathbf{S}_3^T - \mathbf{R}_6^T\tilde{H} & \mathbf{R}_5^T \\ \mathbf{S}_3 - \tilde{H}^T\mathbf{R}_6 & \mathbf{Q} - \mathbf{S}_2\tilde{H} - \tilde{H}^T\mathbf{S}_2^T + \tilde{H}^T\mathbf{R}_2\tilde{H} + F_c^T R_c F_c & \mathbf{S}_1 - F_c^T S_c^T - \tilde{H}^T\mathbf{R}_4^T \\ \mathbf{R}_5 & \mathbf{S}_1^T - S_c F_c - \mathbf{R}_4\tilde{H} & \mathbf{R}_1 + Q_c \end{bmatrix} \le 0 \tag{48}$$

is satisfied for some scalar $\alpha > 0$, then $\Sigma_{OP} : d \to y^*$ has finite L_2 gain less than or equal to α, i.e.,

$$\|y_\tau^*\|_{L_2} \le \gamma \|d_\tau\|_{L_2} \quad \text{and} \quad \gamma \le \alpha \tag{49}$$

Proof. Consider the result in Theorem 4. Since $y^* = V\hat{y} = VF_c y$, we have that the following identity holds:

$$y^T F_c^T V^T V F_c y - y^{*T} y^* = 0 \tag{50}$$

Thus, the dissipation inequality (44) in Theorem 4 can be rewritten as follows:

$$\frac{d\hat{\phi}(\hat{x})}{dt} \le -\begin{bmatrix} y^T & y_c^T \end{bmatrix} \hat{Q} \begin{bmatrix} y \\ y_c \end{bmatrix} + 2\begin{bmatrix} y^T & y_c^T \end{bmatrix} \begin{bmatrix} \mathbf{S}_3 - \tilde{H}^T\mathbf{R}_6 \\ \mathbf{R}_5 \end{bmatrix} d + d^T\mathbf{R}_3 d - y^{*T} y^* \tag{51}$$

where:

$$\hat{Q} = -\begin{bmatrix} \mathbf{Q} - \mathbf{S}_2\tilde{H} - \tilde{H}^T\mathbf{S}_2^T + \tilde{H}^T\mathbf{R}_2\tilde{H} + F_c^T R_c F_c + F_c^T V^T V F_c & \mathbf{S}_1 - F_c^T S_c^T - \tilde{H}^T\mathbf{R}_4^T \\ \mathbf{S}_1^T - S_c F_c - \mathbf{R}_4\tilde{H} & \mathbf{R}_1 + Q_c \end{bmatrix} \tag{52}$$

Assume that $\hat{Q}$ aforementioned is positive definite, (i.e. $\hat{Q} > 0$). Notice that if $\hat{Q} > 0$ then the nonlinear process network in closed-loop with the plant-wide controller Σ_c is asymptotically stable (see Theorem 4). One can now complete squares in (51) and obtain:

$$\frac{d\hat{\phi}(\hat{x})}{dt} \le -\left\| \hat{Q}^{1/2} \begin{bmatrix} y \\ y_c \end{bmatrix} - \hat{S}d \right\|^2 - y^{*T} y^* + d^T(\mathbf{R}_3 + \hat{S}^T\hat{S})d \tag{53}$$

where:

$$\hat{S} = \hat{Q}^{-1/2} \begin{bmatrix} \mathbf{S}_3 - \tilde{H}^T\mathbf{R}_6 \\ \mathbf{R}_5 \end{bmatrix} \tag{54}$$

Thus the process network with inputs $d(t)$ and outputs the vector $y^*(t)$ of variables relevant to dynamic operability is dissipative and satisfies the dissipation inequality:

$$\frac{d\hat{\phi}(\hat{x})}{dt} \le -y^{*T} y^* + d^T(\mathbf{R}_3 + \hat{S}^T\hat{S})d \tag{55}$$

Using the result in Theorem 3 it can be seen that $\Sigma_{OP} : d \to y^*$ has finite L_2 gain γ, with γ less than or equal to $\alpha > 0$ satisfying the matrix inequality:

$$\mathbf{R}_3 + \hat{S}^T\hat{S} - \alpha^2 I \le 0 \tag{56}$$

By means of the Schur complement[39] we have that the aforementioned inequality plus the condition $\hat{Q} > 0$ is equivalent to the LMI in (48). This concludes the proof. ■

The aforementioned theorem establishes an efficient computational procedure to calculate an upper bound (the nonnegative scalar α) for the finite L_2 gain of the nonlinear process network $\Sigma_{OP} : d \to y^*$. The truncated L_2 norm of the disturbance can be estimated from historical plant operation data:

$$\|d_\tau\|_{L_2} = \sqrt{\int_0^\tau d^T(t) d(t) dt} \tag{57}$$

A small value of α indicates good operability in terms of disturbance attenuation by a feedback controller, as it implies smaller disturbance effect on the controlled variables.

Using the result in Theorem 5, the achievable disturbance attenuation of the nonlinear process network can be quantified via the following optimization problem:

Problem 2 (Achievable disturbance attenuation for nonlinear process network).

Given:

(1) *A nonlinear process network (as shown in Figure 2) satisfying the dissipation inequality (38):*

$$\frac{d\phi(x)}{dt} \leq y^T(\mathbf{Q} - \mathbf{S}_2\tilde{H} - \tilde{H}^T\mathbf{S}_2^T + \tilde{H}^T\mathbf{R}_2\tilde{H})y + 2y^T[\mathbf{S}_1 - \tilde{H}^T\mathbf{R}_4^T \quad \mathbf{S}_3 - \tilde{H}^T\mathbf{R}_6]\begin{bmatrix}\hat{u} \\ d\end{bmatrix} + [\hat{u}^T \quad d^T]\begin{bmatrix}\mathbf{R}_1 & \mathbf{R}_5 \\ \mathbf{R}_5^T & \mathbf{R}_3\end{bmatrix}\begin{bmatrix}\hat{u} \\ d\end{bmatrix}$$

(2) *A* (Q_c, S_c, R_c)*-dissipative plant-wide control system* $\Sigma_c : u_c \rightarrow y_c$ *with direct feed-through matrix* J *[as shown in (45)].*

Compute:

$$\min_{Q_c, S_c, R_c, J} \alpha^2$$

subject to matrix inequalities (46) and (48).

The parameter α represents an upper bound for the L_2 gain of the system from the disturbance d to controlled output y^*, which quantifies the effect of disturbance on the controlled output. Therefore, with the value of α, the upper bound of the truncated L_2 norm of y^* (indeed the truncated L_2 norm of the error, with the assumption of $r = 0$) can be calculated from the truncated L_2 norm of the disturbance d [which can be estimated using Eq. 57]. If such an upper bound on the error meets the operational requirement, the plant-wide operability is deemed satisfactory.

Discussion

We have shown that dissipative systems theory can provide operability analysis tools for complex and large-scale nonlinear process networks. The proposed analysis approach can be carried out efficiently with very limited computation and is scalable. Indeed, for a fixed feed-through matrix J in the plant-wide controller Σ_c, Problem 2 reduces to a convex optimization problem with a guaranteed unique solution and can be solved using standard semi-definite programming algorithms. Observe that this characteristic is independent from the actual degree of nonlinearity of the entire process network.

One of the key issues in the proposed approach is to determine the dissipativity of process units. Although Theorem 1 provides an approach to assess dissipativity for general control-affine systems, the dissipativity of many chemical processes can be determined based on mass and energy balances and thermodynamics by using physically motivated storage functions. Ydstie et al. (See Refs. 31 and 32) have linked process thermodynamics with dissipativity. Hangos et al.[35] also developed principles of constructing a Hamiltonian system model for processes so that their dissipativity can be easily determined. These recent developments have made dissipativity based operability analysis a viable approach for chemical engineers.

Multi-loop, multi-unit and plant-wide multivariable control structures are often implemented in the process industries. Multi-loop control employs multiple single loop controllers to control a multivariable process. Multi-unit control refers to the control structure that implements one multivariable controllers for each process unit (or part of it) but ignores the interactions between process units. Generally speaking, multi-loop and multi-unit control is simpler to implement and more fault tolerant but usually achieves poorer control performance than multivariable control. Control structures can be taken into account in the operability analysis so that the impact of control structures can be evaluated. By restricting the structures of matrices Q_c, S_c and R_c in Problem 1 and Problem 2, we can determine whether a plant-wide process can be stabilized by multi-loop, multi-unit or multivariable controllers and what level of control performance can be achieved by different control structures. Matrices Q_c, S_c and R_c should be diagonal, block diagonal and full for multi-loop, multi-unit and plant-wide multivariable control structures, respectively.

Sometimes, it is useful to determine the aforementioned plant-wide operability with controller outputs constraints (or equivalently, the physical limitations of actuators). Assume that a set of controller outputs $y_c^*(t) = V_c y_c(t)$ is constrained. Then the limitation on the controller outputs can be represented as an upper bound on the L_2 gain (denoted as γ_c) of the system from the disturbance $d(t)$ to the controller output $y_c^*(t)$. The condition given in Proposition A.1 (see the Appendix), similar to Theorem 5, can be used to represent this constraint. This condition can then be included as one additional constraint in Problem 1 and Problem 2 to obtain the stabilizability and achievable disturbance attenuation with constrained controller outputs.

Recently, other researchers have also proposed to use dissipative systems theory to analyze complex process networks. Jillson and Ydstie,[40] for example, have proposed a framework for the design of a distributed control system for nonlinear process networks which are assumed to satisfy the first and second laws of thermodynamics. They combine dissipative systems theory with network theory and thermodynamics to derive sufficient conditions for stabilizability using decentralized control, resulting in a scalable approach. Compared with,[40] the approach developed in this article explicitly takes into account the interconnection structure (i.e., network topology) of process networks and can be potentially less conservative by trading off dissipativity of different process units in the network.

Case study

Heat Exchanger Network In this section we study the operability of a heat exchanger network. Consider a particular heat exchanger network as shown in Figures 4 and 5. The notation given in Figure 5 is in accordance to that proposed in the previous section regarding the representation of process networks. The heat exchanger network (HEN) consists of three process to process heat exchangers and one

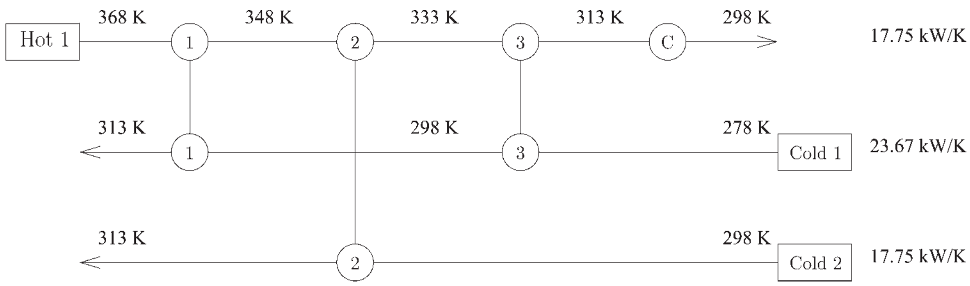

Figure 4. Heat exchanger network case study.

utility heat exchanger labeled ‘C’. There is one hot process stream and two cold process streams in the HEN. The flow rates of these streams are assumed to be constant. Two of the process to process heat exchangers are designed with by-pass streams for control purpose. The utility heat exchanger is controlled by manipulating the flow rate of the cooling water.

The operability study conducted in this case study examines whether the HEN can be stabilized using a simple multi-loop PI-controller and the disturbance propagation can be attenuated. A disturbance is introduced via the second heat exchanger as an L_2 norm-bounded signal of the inlet flow which is denoted as $d_{1,2}$. As can be observed in Figure 5, the disturbance will propagate throughout the entire heat exchanger network due to the interconnection topology of the network.[1] Theorem 5 can be used to quantify the disturbance effects on each of the controlled variables.

It is important to show the dissipativity of each heat exchanger as a subsystem of the entire HEN. The following subsections describe in detail how to establish the (Q, S, R)-dissipativity of each heat exchanger using Theorem 1 and also the (Q, S, R)-dissipativity of a multi-loop PI-controller. The equations of the heat exchangers were obtained based on the modeling approach by Hangos et al.[41]

Heat Exchanger with By-Pass. Consider heat exchangers 1 and 3 from Figure 5 with by-pass in one of the process streams. The physics of the process can be described by the following state space equations:

$$\begin{aligned} \frac{dT_2(t)}{dt} &= \frac{\varepsilon(t)v_1^*}{V_a}(T_1^* - T_2(t)) + \frac{UA}{c_{pa}\rho_a V_a}(T_4(t) - T_2(t)) \\ \frac{dT_4(t)}{dt} &= \frac{v_3^*}{V_b}(T_3(t) - T_4(t)) + \frac{UA}{c_{pb}\rho_b V_b}(T_2(t) - T_4(t)) \end{aligned} \tag{58}$$

where it has been assumed that the heat exchanger has constant mass hold-up and that the input temperature $T_1(t)$ of the hot stream is constant and equal to T_1^*. $\varepsilon(t)v_1^*$ is the fraction of the hot stream flow rate that enters the heat exchanger while $(1 - \varepsilon(t))v_1^*$ is the fraction by-passed to the output. Observe that the fraction $\varepsilon(t)$ is time varying and it is indeed one of the inputs to the heat exchanger:

$$u(t) = \begin{bmatrix} \varepsilon(t) \\ v_3^* T_3(t) \end{bmatrix} \tag{59}$$

The outputs of the heat exchanger are assumed to be:

$$y(t) = \begin{bmatrix} v_1^*(1 - \varepsilon(t))T_1^* + v_1^*\varepsilon(t)T_2(t) \\ v_3^* T_4(t) \end{bmatrix} \tag{60}$$

Consider the following constant parameters:

$$\begin{aligned} \theta_a &= \frac{1}{V_a} \\ \theta_b &= \frac{1}{V_b} \\ \lambda_a &= \frac{UA}{c_{pa}\rho_a V_a} \\ \lambda_b &= \frac{UA}{c_{pb}\rho_b V_b} \end{aligned} \tag{61}$$

and the deviation variables:

$$\begin{aligned} \delta T_2 &= T_2 - T_2^* \\ \delta T_3 &= T_3 - T_3^* \\ \delta T_4 &= T_4 - T_4^* \\ \delta\varepsilon &= \varepsilon - \varepsilon^* \end{aligned} \tag{62}$$

where, to simplify the notation, we have dropped the explicit time dependence of the variables. The tuple $(T_2^*, T_3^*, T_4^*, \varepsilon^*)$ defines the operating point of interest. With the aforemen-

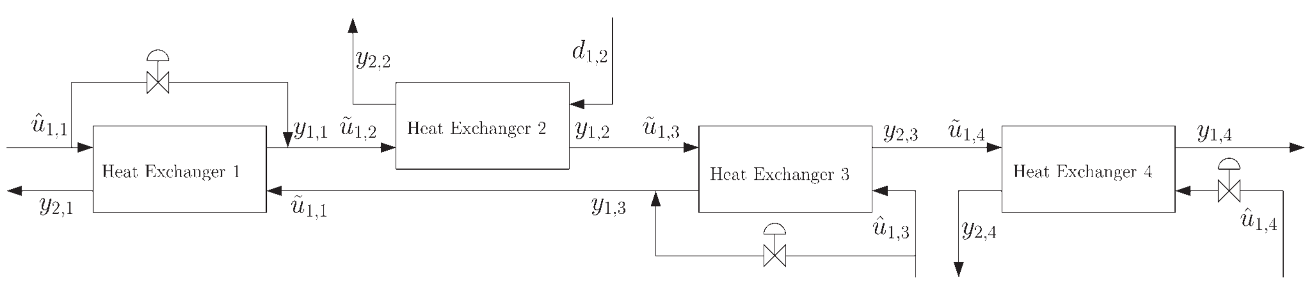

Figure 5. Heat exchanger network case study with control.

tioned definitions, the heat exchanger model (58)–(60) can be written as follows:

$$\begin{aligned}\delta\dot{T}_2 &= -(\lambda_a + \theta_a v_1^*\varepsilon^*)\delta T_2 + \lambda_a\delta T_4 + \theta_a(T_1^* - T_2^* - \delta T_2)v_1^*\ \delta\varepsilon\\ \delta\dot{T}_4 &= \lambda_b\delta T_2 - (\lambda_b + \theta_b v_3^*)\delta T_4 + \theta_b v_3^*\ \delta T_3\\ y_1 &= v_1^*\varepsilon^*\delta T_2 - v_1^*(T_1^* - T_2^* - \delta T_2)\delta\varepsilon\\ y_2 &= v_3^*\delta T_4\end{aligned} \tag{63}$$

Define the input vector as:

$$u = \begin{bmatrix}\delta\varepsilon\\ v_3^*\delta T_3\end{bmatrix} \tag{64}$$

Then, the heat exchanger nonlinear model (63) has the form:

$$P: \begin{cases}\dot{x} = f(x) + G(x)u\\ y = h(x) + J(x)u\end{cases} \tag{65}$$

where:

$$f(x) = \begin{bmatrix}-(\lambda_a + \theta_a v_1^*\varepsilon^*)\ \delta T_2 + \lambda_a\delta T_4\\ \lambda_b\delta T_2 - (\lambda_b + \theta_b v_3^*)\delta T_4\end{bmatrix} \quad G(x) = \begin{bmatrix}\theta_a(T_1^* - T_2^* - \delta T_2)v_1^* & 0\\ 0 & \theta_b\end{bmatrix}$$

$$h(x) = \begin{bmatrix}v_1^*\varepsilon^*\delta T_2\\ v_3^*\delta T_4\end{bmatrix} \quad J(x) = \begin{bmatrix}-v_1^*(T_1^* - T_2^* - \delta T_2) & 0\\ 0 & 0\end{bmatrix} \tag{66}$$

It is found that the heat exchangers 1 and 3 as depicted in Figure 5 with state space nonlinear model (65)–(66) are (Q,S,R)-dissipative with $Q < 0$. Details can be found in Proposition A.2.

Utility Heat Exchanger. Consider the utility heat exchanger numbered as 4 in Figure 5. The outlet temperature of the process stream is to be controlled by manipulating the flow rate of the cooling medium (water). The system behavior can therefore be described using the following equations:

$$\begin{aligned}\frac{dT_2(t)}{dt} &= \frac{v_1^*}{V_a}(T_1(t) - T_2(t)) + \frac{UA}{c_{pa}\rho_a V_a}(T_4(t) - T_2(t))\\ \frac{dT_4(t)}{dt} &= \frac{v_3(t)}{V_b}(T_3^* - T_4(t)) + \frac{UA}{c_{pb}\rho_b V_b}(T_2(t) - T_4(t))\end{aligned} \tag{67}$$

It has been assumed that the heat exchanger has constant mass hold-up, the flow rate of the process stream $v_1(t)$ and supply temperature of the cooling water $T_3(t)$ are constant, corresponding to v_1^* and T_3^*. Equation 67 can be rewritten in its deviation variables as follows:

$$\begin{aligned}\delta\dot{T}_2 &= -(\lambda_a + \theta_a v_1^*)\delta T_2 + \lambda_a\delta T_4 + \theta_a v_1^*\ \delta T_1\\ \delta\dot{T}_4 &= \lambda_b\delta T_2 - (\lambda_b + \theta_b v_3^*)\delta T_4 + \theta_b(T_3^* - T_4^* - \delta T_4)\delta v_3\\ y_1 &= v_1^*\delta T_2\\ y_2 &= v_3^*\delta T_4 + (T_4^* + \delta T_4)\delta v_3\end{aligned} \tag{68}$$

with the input vector:

$$u = \begin{bmatrix}v_1^*\ \delta T_1\\ \delta v_3\end{bmatrix} \tag{69}$$

Heat exchanger 4 as depicted in Figure 5 with state space nonlinear model (68), (69) can be shown to be (Q,S,R)-dissipative with $Q < 0$. Details can be found in Proposition A.3.

Common Process to Process Heat Exchanger. Consider heat exchanger 2 in Figure 5 where the flow rates of both streams are constant while the inlet temperatures may vary. The system can be written as follows:

$$\begin{aligned}\frac{dT_2(t)}{dt} &= \frac{v_1^*}{V_a}(T_1(t) - T_2(t)) + \frac{UA}{c_{pa}\rho_a V_a}(T_4(t) - T_2(t))\\ \frac{dT_4(t)}{dt} &= \frac{v_3^*}{V_b}(T_3(t) - T_4(t)) + \frac{UA}{c_{pb}\rho_b V_b}(T_2(t) - T_4(t))\end{aligned} \tag{70}$$

It is assumed that the heat exchanger has constant mass hold-up, the flow rates of both process streams $v_1(t)$ and $V_3(t)$ are constant, corresponding to v_1^* and v_3^*. The heat exchanger description in Eq. 70 can be rewritten in terms of deviation variables as follows:

$$\begin{aligned}\delta\dot{T}_2 &= -(\lambda_a + \theta_a v_1^*)\delta T_2 + \lambda_a\delta T_4 + \theta_a v_1^*\ \delta T_1\\ \delta\dot{T}_4 &= \lambda_b\delta T_2 - (\lambda_b + \theta_b v_3^*)\delta T_4 + \theta_b v_3^*\ \delta T_3\\ y_1 &= v_1^*\ \delta T_2\\ y_2 &= v_3^*\ \delta T_4\end{aligned} \tag{71}$$

with the input vector:

$$u = \begin{bmatrix}v_1^*\ \delta T_1\\ v_3^*\ \delta T_3\end{bmatrix} \tag{72}$$

It must be noted in this case the heat exchanger described in (71) and (72) is linear in the form of:

$$P: \begin{cases}\dot{x} = Ax + Bu\\ y = Cx + Du\end{cases} \tag{73}$$

Consider heat exchanger 2 as depicted in Figure 5 with state space linear model. It can be shown to be (Q,S,R)-dissipative with $Q < 0$. Details can be found in Proposition A.4.

Proportional and Integral Controller In the case study aforementioned, the operability study is done using multi-loop PI-controller. The following state space representation is used to describe a multi-loop PI-controller:

$$\begin{aligned} \dot{x} &= K_I u_c \\ y_c &= x + K_c u_c \end{aligned} \tag{74}$$

As can be seen from Eq. 74 a PI controller has a direct feed-through matrix $J = K_c$.

Proposition 2 (*(Q, S, R)-dissipativity of PI controller*). *A multi-loop proportional and integral (PI) controller with $K_c K_I > 0$ is dissipative with respect to the following Q, S and R*:

$$Q = 0 \qquad S = P\, K_I \qquad R = -K_C^T\, PK_I \tag{75}$$

Proof. Consider the following *storage function* $\phi(x) : X \rightarrow \mathrm{R}^+$:

$$\phi(x) = x^T\, Px \tag{76}$$

where P is symmetric. Then, the system in (74) has the following supply rate:

$$\begin{aligned} \dot{\phi}(x) &= \dot{x}^T Px + x^T P\dot{x} \\ &= (K_I u)^T Px + x^T PK_I u \\ &= (K_I u)^T P(y - K_c u) + (y - K_c u)^T P(K_I u) \\ &= 2y^T PK_I u - 2u^T K_c^T PK_I u \\ &\leq 2y^T PK_I u - u^T K_c^T PK_I u \end{aligned} \tag{77}$$

Therefore, the PI-controller in (74) is dissipative with:

$$Q = 0 \qquad S = P\, K_I \qquad R = -K_C^T\, PK_I \tag{78}$$

■

Heat Exchanger Network Operability Study. The (Q, S, R)-dissipativity of each heat exchanger as subsystems and the multi-loop PI-controller have been established as shown. The next step involves the formulation of the network description considering the topology of all the heat exchangers. Following this, the stability, stabilizability and disturbance attenuation analysis can be conducted.

Stability and stabilizability

The interaction between units of the heat exchanger network in Figure 5 is represented by the following *interconnection matrix* $\tilde{H}$ [as defined in (4)]:

$$\begin{bmatrix} \tilde{u}_{1,1} \\ \tilde{u}_{1,2} \\ \tilde{u}_{1,3} \\ \tilde{u}_{1,4} \end{bmatrix} = - \begin{bmatrix} 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} y_{1,1} \\ y_{2,1} \\ y_{1,2} \\ y_{2,2} \\ y_{1,3} \\ y_{2,3} \\ y_{1,4} \\ y_{2,4} \end{bmatrix} \tag{79}$$

Following this, the overall open-loop (Q_{OP}, S_{OP}, R_{OP})-dissipativity of the HEN can be calculated according to Eq. 38.

As Q_{OP} is not negative definite (from (132) in the Appendix), the stability of the plant-wide open-loop process cannot be determined. However, we can assess the stabilizability by using Theorem 4. To incorporate the controller into the system, a *selector matrix F_c* is employed (as shown below), as described in Eq. 41:

$$\begin{bmatrix} u_{c1,1} \\ u_{c1,3} \\ u_{c1,4} \end{bmatrix} = -F_c \begin{bmatrix} y_{1,1} \\ y_{2,1} \\ y_{1,2} \\ y_{2,2} \\ y_{1,3} \\ y_{2,3} \\ y_{1,4} \\ y_{2,4} \end{bmatrix} \tag{80}$$

where:

$$F_c = \begin{bmatrix} \frac{1}{v_{1,1}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{v_{1,3}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{v_{1,4}} & 0 \end{bmatrix} \tag{81}$$

Assume PI-controllers are employed. That is, $Q_c = 0$. By solving Problem 1, it can be shown that the plant-wide process can be stabilized with controllers satisfying the following (Q_c, S_c, R_c)-dissipativity

$$S_c = \begin{bmatrix} -17.4 & 0 & 0 \\ 0 & 12.8 & 0 \\ 0 & 0 & -1.46 \times 10^5 \end{bmatrix},$$
$$R_c = \begin{bmatrix} -4.47 \times 10^8 & 0 & 0 \\ 0 & -4.47 \times 10^8 & 0 \\ 0 & 0 & -4.47 \times 10^8 \end{bmatrix} \tag{82}$$

The closed-loop system is shown to be (Q_{cl}, S_{cl}, R_{cl})-dissipative. Moreover, Q_{cl} is negative definite (Eq. 135 in the Appendix). This indicates that the heat exchanger network in this case study can be stabilized using a multi-loop PI-controller.

Achievable disturbance attenuation

The disturbance attenuation analysis on one of the controlled outputs was further investigated. In this instance, the final outlet temperature of the hot stream $(\hat{y}_{1,4})$ in Figure 5 is considered to be the controlled output of interest and the product of the flow rate and temperature of the cold stream to the second heat exchanger $(d_{1,2})$ as the disturbance variable. By solving Problem 2, the effect of disturbance to the output of interest can be quantified. It was found that the solution to the problem results in the same multi-loop PI controller as calculated previously.

It was found that the L_2 gain between $d_{1,2}$ and $\hat{y}_{1,4}$ is less than or equal to $\alpha = 88.5\ m^{-3}\ s$. This value of α represents the L_2 gain between the variation of $d_{1,2}$ in the second heat exchanger to the controlled temperature $\hat{y}_{1,4}$. The actual effect of disturbance to the controlled temperature is indicated by a normalized value of α, denoted $\tilde{\alpha}$, which represents the L_2 gain between the variation of temperature only (ignoring the effect of flowrate) in $d_{1,2}$ and the controlled temperature $\hat{y}_{1,4}$. The value of $\tilde{\alpha}$ can be easily calculated because the inlet flow rate of $d_{1,2}$ is assumed constant. $\tilde{\alpha}$ was found to be 0.374, which means that with the control system specified, the magnitude of deviation of the disturbance can be attenuated such that its effect on the controlled variable is at most 37% (in terms of the truncated L_2 norm).

Conclusion

In this article, a new operability analysis approach for nonlinear plant-wide processes is developed from a network perspective. There are two distinctive features of the proposed approach. First, a new framework for representing process units is developed to model separately the physical mass and energy flows from/to process units and the information flows between process units and control systems. This leads to straightforward description of process network topologies directly from process flowsheets. Second, the concept of dissipativity and dissipative theory are used as the theoretical foundation of this work as they are most appropriate for dynamical analysis for interconnected nonlinear systems. Operability analysis methods have been developed to determine the open-loop plant-wide stability, stabilizability and achievable disturbance attenuation from the dissipativity of process units and the topology of the process network. This analysis approach is very scalable as it is not based on the model of the entire plant-wide process and thus can be used to analyze large process networks. This work shows the links between process dissipativity and operability, and provides new insights into how interconnections between process units affect plant-wide operability. As dissipativity of chemical processes can be linked to their thermodynamic properties and determined based on physically motivated storage functions, the proposed operability analysis is a viable approach for chemical engineers.

Acknowledgements

The support of the Australian Research Council (Grant DP0558755) is gratefully acknowledged. The second author wishes to acknowledge the finanical support from the University of New South Wales Postgraduate Award.

Literature Cited

1. Luyben WL, Tyréus BD, Luyben ML. *Plant-Wide Process Control*. New York: McGraw-Hill, 1999.
2. Perkins JD. The integration of design and control: the key to future processing systems? In: The Proceedings of the Sixth World Congress of Chemical Engineering; Sep 23–27, 2001, Melbourne, Australia.
3. Chen R, McAvoy T, Zafiriou E. Plant-wide control system design: extension to multiple-forcing and multiple steady-state operation. *Ind Eng Chem Res*. 2004;43:3685–3694.
4. Barton GW, Chan WK, Perkins JD. Interaction between process design and process control: the role of open-loop indicators. *J Process Control*. 1991;1:161–170.
5. Morari M. Design of resilient processing plant. III. A general framework for the assessment of dynamic resilience. *Chem Eng Sci*. 1983;38:1881–1891.
6. Bahri PA, Bandoni J, Romagnoli J. Integrated flexibility and controllability analysis in design of chemical processes. *AIChE J*. 1997;42:997–1015.
7. Perkins JD, Walsh SPK. Optimization as a tool for design/control integration. *Comput Chem Eng*. 1996;20:315–323.
8. Kookos IK, Perkins JD. An algorithm for simultaneous process design and control. *Ind Eng Chem Res*. 2001;40:4079–4088.
9. Bansal V, Sakizlis V, Ross R, Perkins JD, Pistikopoulos EN. New algorithms for mixed-integer dynamic optimization. *Comput Chem Eng*. 2003;27:647–668.
10. Sakizlis V, Perkins JD, Pistikopoulos EN. Recent advances in optimization-based simultaneous process and control design. *Comput Chem Eng*. 2004;28:2069–2086.
11. Vinson DR, Georgakis C. A new measure of process output controllability. *J Process Control*. 2000;10:185–194.
12. Hernjak N, Doyle FJ III, Ogunnaike BA, Pearson RK. Chemical process characterization for control design. In: Seferlis P, Georgiadis MC, editors. *The Integration of Process Design and Control, Computer-Aided Chemical Engineering*, vol. 17. Amsterdam: Elsevier BV, 2004:42–75, Chapter A2.
13. Seider WD, Brengel DD, Widagdo S. Nonlinear-analysis in process design. *AIChE J*. 1991;37:1–38.
14. Russo LP, Bequette BW. Impact of process design on the multiplicity behavior of a jacketed exothermic CSTR. *AIChE J*. 1995; 41: 135–147.
15. Ray WH, Villa CM. Nonlinear dynamics found in polymerisation processes. *Chem Eng Sci*. 2000;55:275–290.
16. Dorn C, Morari M. Qualitative analysis for homogeneous azeotropic distillation: 2. Bifurcation analysis. *Ind Eng Chem Res*. 2002;41: 3943–3962.
17. Marquardt W, Mönnigmann M. Constructive nonlinear dynamics in process systems engineering. *Comput Chem Eng*. 2005;29:1265–1275.
18. Hangos KM, Alonso AA, Perkins JD, Ydstie BE. Thermodynamic approach to the structural stability of process plants. *AIChE J*. 1999;45:802–816.
19. Rojas OJ, Bao J, Lee PL. Linear control of nonlinear processes: the regions of steady-state attainability. *Ind Eng Chem Res*. 2006; 45:7552–7565.
20. Rojas OJ, Bao J, Lee PL. A dynamic operability analysis approach for nonlinear processes. *J Process Control*. 2007;17:157–172.
21. Rojas OJ, Bao J, Lee PL. On dissipativity, passivity and dynamic operability of nonlinear processes. *J Process Control*. 2008; 18:515–526.
22. Skogestad S. Control structure design for complete chemical plants. *Comput Chem Eng*. 2004;28:219–234.
23. Stephanopoulos G, Ng C. Perspectives on the synthesis of plant-wide control structures. *J Process Control*. 2000;10:97–111.
24. Larsson T, Skogestad S. Plant-wide control—a review and a new design procedure. *Model Identification Control*. 2000;21:209–240.
25. Ydstie BE. New vistas for process control: integrating physics and communication networks. *AIChE J*. 2002;48:422–426.
26. Willems JC. Dissipative dynamical systems. Part I. General theory. *Arch Ration Mech Anal*. 1972;45:321–351.
27. Willems JC. Dissipative dynamical systems. Part II. Linear systems with quadratic supply rates. *Arch Ration Mech Anal*. 1972;45:352–393.
28. Kokotovic P, Arcak M. Constructive nonlinear control: a historical perspective. *Automatica*. 2001;37:637–662.
29. Sepulchre R, Jancovic M, Kokotovic P. *Constructive Nonlinear Control*. London: Springer-Verlag, 1997.
30. Desoer CA, Vidyasagar M. *Feedback Systems: Input-Output Properties*. New York: Academic Press, 1975.
31. Alonso AA, Ydstie BE. Process systems, passivity and the second law of thermodynamics. *Comput Chem Eng*. 1996;2:1119–1124.
32. Ydstie BE, Alonso AA. Process systems, and passivity via the Clausius-Planck inequality. *Syst Control Lett*. 1997;30:253–264.
33. Farschman CA, Viswanath KP, Ydstie BE. Process systems and inventory control. *AIChE J*. 1998;44:1841–1857.
34. Ydstie BE. Passivity based control via the second law. *Comput Chem Eng*. 2002;26:1037–1048.
35. Hangos KM, Bokor J, Szederkényi G. Hamiltonian view of process systems. *AIChE J*. 2001;47:1819–1813.
36. Hill DJ, Moylan PJ. The stability of nonlinear dissipative systems. *IEEE Trans Automatic Control*. 1976;21:708–711.
37. Moylan PJ, Hill DJ. Stability criteria for large-scale systems. *IEEE Trans Automatic Control*. 1978;23:143–149.
38. Khalil HK. *Nonlinear Systems*, 3rd ed. Upper Saddle River, New Jersey: Prentice Hall, 2002.
39. Boyd S, Ghaoui LE, Feron E, Balakrishnan V. *Linear Matrix Inequalities in System and Control Theory*. Philadelphia: SIAM, 1994.

40. Jillson KR, Ydstie BE. Process networks with decentralized inventory and flow control. *J Process Control*. 2007;17:399–413.
41. Hangos KM, Bokor J, Szederkényi G. *Analysis and Control of Nonlinear Process Systems*. London: Springer-Verlag, 2004.
42. Hill DJ, Moylan PJ. Dissipative dynamical systems: basic input-output and state properties. *J Franklin Inst*. 1980;309:327–357.
43. Hill DJ, Moylan PJ. Stability results for nonlinear feedback systems. *Automatica*. 1977;13:377–382.

Appendix

Relevant definitions and theorems

Main Definitions and Results of Dissipative Systems Theory. In this section some of the main concepts and definitions of dissipative systems theory are reviewed. This material is mainly adopted from[26,42] and the readers are referred to those sources for further details.

Definition A.1 (Dynamical systems).[26] *A dynamical system Σ is defined through the input space U, output space Y, state space X and two functions: the state transition function $\psi(t, t_0, x_0, u)$ which maps $\mathrm{R}_2^+ \times X \times U$ into X, and the read-out function $r(\psi(t, t_0, x_0, u), u(t))$ which maps $X \times U$ into Y, where* $\mathrm{R}_2^+ = \{(t_2, t_1) | t_2 \geq t_1 \text{ and } t_1, t_2 \in \mathrm{R}\}$.

The input and output space U and Y are closed under the shift operator, i.e., if $u \in U$, $y \in Y$, then functions $u(t+T)$ and $y(t+T)$ belong to U and Y respectively for any $T \in$ R.

The state transition function $\psi(t, t_0, x_0, u)$ is a map that obeys the following conditions:

(1) *Consistency: $\psi(t_0, t_0, x_0, u) = x_0$ for all $t_0 \in$* R, *$x_0 \in X$ and $u \in U$;*

(2) *Determinism: $\psi(t_1, t_0, x_0, u_1) = \psi(t_1, t_0, x_0, u_2)$ for all $(t_1, t_0) \in \mathrm{R}_2^+$ and $x_0 \in X$ and $u_1, u_2 \in U$ satisfying $u_1(t) = u_2(t)$ for all $t_0 \leq t \leq t_1$.*

(3) *Semi-group property: $\psi(t_2, t_0, x_0, u) = \psi(t_2, t_1, \psi(t_1, t_0, x_0, u), u)$ for all $t_0 \leq t_1 \leq t_2$, $x_0 \in X$ and $u \in U$.*

(4) *Stationarity: $\psi(t_1+T, t_0+T, x_0, u_T) = \psi(t_1, t_0, x_0, u)$ for all $(t_1, t_0) \in \mathrm{R}_2^+$, $T \in$* R *and $x_0 \in X$ and $u, u_T \in U$ related by $u_T(t) = u(t+T)$ for all $t \in$* R.

There exists an element $y \in Y$ such that $y(t) = r(\psi(t, t_0, x_0, u), u(t))$ for all $t \geq t_0$.

Definition A.2 (Supply rate).[26] *The supply rate is a real valued function:*

$$w(u, y) : U \times Y \rightarrow \mathrm{R} \tag{83}$$

which is locally integrable for any $u \in U$, $y \in Y$ and $\tau \geq 0$. That is,

$$\int_0^\tau |w(u(t), y(t))| dtt \ < \ \infty \tag{84}$$

Definition A.3 (Dissipative dynamical system).[26] *A dynamic system Σ with supply rate $w(u, y)$ is said to be dissipative if there exists a nonnegative function $\phi(x) : X \rightarrow \mathrm{R}^+$, called the storage function, such that for all initial condition $x_0 \in X$, input $u \in U$ and $\tau \geq 0$,*

$$\phi(x) - \phi(x_0) \leq \int_0^\tau w(u(t), y(t)) dt \tag{85}$$

where x is the state at time $t = \tau$.

Hill and Moylan[36,42,43] developed a comprehensive analytical framework for those dissipative systems that posses a quadratic supply rate of the form:

$$w(u, y) = y^T Q y + 2 y^T S u + u^T R u \tag{86}$$

where $Q \in \mathbf{R}^{p \times p}$, $S \in \mathbf{R}^{p \times m}$, $R \in \mathbf{R}^{m \times m}$ are constant matrices with Q and R symmetric. These type of systems are said to be (Q, S, R)-dissipative to emphasize the special structure of the associated supply rate.

Theorem of (Q,S,R)*-dissipative system synthesis*

Theorem A.1 *(Synthesis of (Q, S, R)-dissipative systems).*[21] *Consider a nonlinear free system*

$$\dot{x} = f(x, 0) \tag{87}$$

with an asymptotically stable equilibrium $x = 0$ and a known Lyapunov function $\phi(x) : \mathbf{R}^n \rightarrow \mathbf{R}^+$ such that $\phi(x) > 0 \ \forall x \neq 0$ and $\phi(0) = 0$. For given matrices $Q \in \mathbf{R}^{p \times p}$, $S \in \mathbf{R}^{p \times m}$ and $R \in \mathbf{R}^{m \times m}$ and a given feed-through matrix $J(x) : \mathbf{R}^n \rightarrow \mathbf{R}^{p \times m}$, there always exists a nonlinear function matrix $G(x) : \mathbf{R}^n \rightarrow \mathbf{R}^{n \times m}$ and a nonlinear read-out map $h(x) : \mathbf{R}^n \rightarrow \mathbf{R}^p$ such that the input-affine nonlinear system is (Q, S, R)-dissipative if

$$R + J(x)^T S + S^T J(x) + J(x)^T Q J(x) \geq 0. \tag{88}$$

On controller output constraint

Proposition A.1 *(Achievable disturbance attenuation with constrained controller outputs).*

Consider the nonlinear process network shown in Figure 2 together with the assumptions described in Theorem 5. In addition, assume that the outputs of the plant-wide controller Σ_c that are constrained are given by:

$$y_c^* = V_c y_c \tag{89}$$

where V_c is a constant selector matrix of appropriate dimension. If the following linear matrix inequality (LMI)

$$\begin{bmatrix} \mathbf{R}_3 - \alpha_c^2 I & \mathbf{S}_3^T - \mathbf{R}_6^T \tilde{H} & \mathbf{R}_5^T \\ \mathbf{S}_3 - \tilde{H}^T \mathbf{R}_6 & \mathbf{Q} - \mathbf{S}_2 \tilde{H} - \tilde{H}^T \mathbf{S}_2^T + \tilde{H}^T \mathbf{R}_2 \tilde{H} + F_c^T R_c F_c & \mathbf{S}_1 - F_c^T S_c^T - \tilde{H}^T \mathbf{R}_4^T \\ \mathbf{R}_5 & \mathbf{S}_1^T - S_c F_c - \mathbf{R}_4 \tilde{H} & \mathbf{R}_1 + Q_c + V_c^T V_c \end{bmatrix} \leq 0 \tag{90}$$

is satisfied for some scalar $\alpha_c > 0$, *then the nonlinear process network* $\bar{\Sigma}_{OP} : d \rightarrow y_c^*$ *has finite* L_2 *gain* γ_c, *with* γ_c *less than or equal to* $\alpha_c > 0$.

Proof. The proof follows similar arguments to those used in the proof of Theorem 5 starting with the identity $\boldsymbol{y}_c^T \boldsymbol{V}_c^T \boldsymbol{V}_c \boldsymbol{y}_c - \boldsymbol{y}_c^{*T} \boldsymbol{y}_c^* = \boldsymbol{0}$.

Dissipativity of the HEN

Proposition A.2 *(Dissipativity of heat exchanger with bypass). The heat exchangers 1 and 3 as depicted in Figure 5 with state space nonlinear model (65), (66) are* (Q, S, R)*-dissipative, with:*

$$Q = \begin{bmatrix} q_1 & q_3 \\ q_3 & q_2 \end{bmatrix} < 0$$
$$S = \begin{bmatrix} s_1 & s_2 \\ s_3 & s_4 \end{bmatrix} \tag{91}$$
$$R = \begin{bmatrix} r_1 & r_3 \\ r_3 & r_2 \end{bmatrix}$$

if δT_2 *is bounded by Inequality (95).*

Proof: Consider the result in Theorem 1. Let the storage function $\phi(x) : \mathrm{R}^n \rightarrow \mathrm{R}^+$ be:

$$\begin{aligned} \phi(x) &= x^T P x \\ &= [\, \delta T_2 \quad \delta T_4 \,] \begin{bmatrix} p_1 & p_3 \\ p_3 & p_2 \end{bmatrix} \begin{bmatrix} \delta T_2 \\ \delta T_4 \end{bmatrix} \end{aligned} \tag{92}$$

Also let:

$$l(x) = \begin{bmatrix} l_1\ \delta T_4 \\ 0 \\ l_2\ \delta T_2 \\ l_3\ \delta T_4 \\ l_4\ \delta T_2 + l_5\ \delta T_4 \end{bmatrix}$$
$$W(x)^T = \begin{bmatrix} \sqrt{\hat{q}_1}\ \delta T_2 & \sqrt{w_4 - 2\hat{q}_1\ \delta T_2^2} & w_3 & w_5 & 0 \\ w_1 & 0 & w_2 & 0 & 0 \end{bmatrix} \tag{93}$$

where $l_1, l_2, l_3, l_4, l_5, w_1, w_2, w_3, w_4, w_5$ are parameters and:

$$\hat{q}_1 = -q_1 (v_1^*)^2 \tag{94}$$

Observe that for the expression (93) to be valid it is required that:

$$\begin{aligned} &\hat{q}_1 > 0 \\ &|\delta T_2| < \sqrt{\frac{w_4}{2\hat{q}_1}} \end{aligned} \tag{95}$$

Let:

$$\begin{aligned} s_1 &= q_1 v_1^* (T_1^* - T_2^*) \\ s_3 &= q_3 v_1^* (T_1^* - T_2^*) \end{aligned} \tag{96}$$

We now check the conditions outlined in Theorem 1: Condition-(ii):

$$\begin{aligned} \frac{\partial \phi^T}{\partial x} f(x) &= 2(-p_1(\lambda_a + \theta_a v_1^* \varepsilon^*) + p_3 \lambda_b) \delta T_2^2 \\ &+ 2(p_1 \lambda_a - p_3(\lambda_a + \theta_a v_1^* \varepsilon^*) + p_2 \lambda_b - p_3(\lambda_b + \theta_b v_3^*)) \delta T_2 \delta T_4 \\ &+ 2(-p_2(\lambda_b + \theta_b v_3^*) + p_3 \lambda_a) \delta T_4^2 \end{aligned} \tag{97}$$

$$\begin{aligned} h(x)^T Q h(x) - l(x)^T l(x) &= (-\hat{q}_1 (\varepsilon^*)^2 - l_2^2 - l_4^2) \delta T_2^2 \\ &+ (2\varepsilon^* q_3 v_1^* v_3^* - 2 l_4 l_5) \delta T_2 \delta T_4 \\ &+ (q_2 (v_3^*)^2 - l_1^2 - l_3^2 - l_5^2) \delta T_4^2 \end{aligned} \tag{98}$$

Equating (97) and (98) and matching the coefficients of the terms with the same power of δT_2 and δT_4, the following set of equations are obtained:

$$\begin{aligned} &2(-p_1(\lambda_a + \theta_a v_1^* \varepsilon^*) + p_3 \lambda_b) = -\hat{q}_1 (\varepsilon^*)^2 - l_2^2 - l_4^2 \\ &p_1 \lambda_a - p_3(\lambda_a + \theta_a v_1^* \varepsilon^*) + p_2 \lambda_b - p_3(\lambda_b + \theta_b v_3^*) = \varepsilon^* q_3 v_1^* v_3^* - l_4 l_5 \\ &2(-p_2(\lambda_b + \theta_b v_3^*) + p_3 \lambda_a) = q_2 (v_3^*)^2 - l_1^2 - l_3^2 - l_5^2 \end{aligned} \tag{99}$$

Condition-(iii):

$$\frac{1}{2} G(x)^T \frac{\partial \phi}{\partial x} = \begin{bmatrix} \theta_a v_1^* p_1 (T_1^* - T_2^* - \delta T_2)\, \delta T_2 + \theta_a v_1^* p_3 (T_1^* - T_2^* - \delta T_2)\, \delta T_4 \\ \theta_b p_3\, \delta T_2 + \theta_b p_2\, \delta T_4 \end{bmatrix} \tag{100}$$

By substituting the values of s_1 and s_3 in (96), the following equations are obtained:

$$(QJ(x) + S)^T h(x) = \begin{bmatrix} -\hat{q}_1 \varepsilon^* \delta T_2^2 + q_3 v_1^* v_3^* \delta T_2 \delta T_4 \\ s_2 v_1^* \varepsilon^*\ \delta T_2 + s_4 v_3^*\ \delta T_4 \end{bmatrix} \tag{101}$$

$$-W(x)^T l(x) = \begin{bmatrix} -\sqrt{\hat{q}_1}\ l_1\ \delta T_2 \delta T_4 - w_3 l_2\ \delta T_2 - w_5 l_3\ \delta T_4 \\ -w_1 l_1\ \delta T_4 - w_2 l_2\ \delta T_2 \end{bmatrix} \tag{102}$$

Next, we match the coefficients of the terms with the same power of δT_2 and δT_4 to obtain the following set of equations:

$$\begin{aligned} \theta_a v_1^* p_1 (T_1^* - T_2^*) &= -w_3 l_2 \\ \theta_a v_1^* p_1 &= \hat{q}_1 \varepsilon^* \\ -\theta_a v_1^* p_3 &= q_3 v_1^* v_3^* - \sqrt{\hat{q}_1}\ l_1 \\ \theta_a v_1^* p_3 (T_1^* - T_2^*) &= -w_5 l_3 \\ \theta_b p_2 &= s_4 v_3^* - w_1 l_1 \\ \theta_b p_3 &= s_2 v_1^* \varepsilon^* - w_2 l_2 \end{aligned} \tag{103}$$

Condition-(iv): Replacing s_1 and s_3 in (96) we obtain:

$$R + J(x)^T S + S^T J(x) + J(x)^T Q J(x) = \begin{bmatrix} r_1 + \hat{q}_1 (T_1^* - T_2^*)^2 - \hat{q}_1\ \delta T_2^2 & r_3 - s_2 v_1^* (T_1^* - T_2^* - \delta T_2) \\ r_3 - s_2 v_1^* (T_1^* - T_2^* - \delta T_2) & r_2 \end{bmatrix} \tag{104}$$

Also:

$$W(x)^T W(x) = \begin{bmatrix} w_4 + w_3^2 + w_5^2 - \hat{q}_1\ \delta T_2^2 & w_2 w_3 + w_1 \sqrt{\hat{q}_1}\ \delta T_2 \\ w_2 w_3 + w_1 \sqrt{\hat{q}_1}\ \delta T_2 & w_1^2 + w_2^2 \end{bmatrix} \tag{105}$$

Equating (104) and (105) and matching the coefficients of the terms with the same power of δT_2, the following set of equations are obtained:

$$\begin{aligned} r_1 + \hat{q}_1 (T_1^* - T_2^*)^2 &= w_4 + w_3^2 + w_5^2 \\ r_3 - s_2 v_1^* (T_1^* - T_2^*) &= w_2 w_3 \\ s_2 v_1^* &= w_1 \sqrt{\hat{q}_1} \\ r_2 &= w_1^2 + w_2^2 \end{aligned} \tag{106}$$

We can now combine the set of Eqs. 99, 103, and 106 to determine the unknown parameters contained in the matrices Q, S, R, $l(x)$ and $W(x)$. There are 23 parameters and 15 equations [including (96)].

Proposition A.3 *(Dissipativity of utility heat exchanger). Heat exchanger 4 as depicted in Figure 5 with state space nonlinear model (68), (69) is (Q, S, R)-dissipative with:*

$$Q = \begin{bmatrix} q_1 & q_3 \\ q_3 & q_2 \end{bmatrix} < 0 \quad S = \begin{bmatrix} s_1 & s_2 \\ s_3 & s_4 \end{bmatrix} \quad R = \begin{bmatrix} r_1 & r_3 \\ r_3 & r_2 \end{bmatrix} \tag{107}$$

if δT_2 is bounded by Inequality (111).

Proof: Consider the result in Theorem 1. Let the storage function $\phi(x) : \mathrm{R}^n \to \mathrm{R}^+$ be:

$$\phi(x) = x^T P x \qquad \phi(x) = [\,\delta T_2 \quad \delta T_4\,] \begin{bmatrix} p_1 & p_3 \\ p_3 & p_2 \end{bmatrix} \begin{bmatrix} \delta T_2 \\ \delta T_4 \end{bmatrix} \tag{108}$$

Also let:

$$l(x) = \begin{bmatrix} l_1\ \delta T_2 \\ 0 \\ l_2\ \delta T_4 \\ l_3\ \delta T_2 \\ l_4\ \delta T_2 + l_5\ \delta T_4 \end{bmatrix} \qquad W(x)^T = \begin{bmatrix} w_1 & 0 & w_2 & 0 & 0 \\ \sqrt{\hat{q}_2}\ \delta T_4 & \sqrt{w_4 - 2\hat{q}_2\ \delta T_4^2} & w_3 & w_5 & 0 \end{bmatrix} \tag{109}$$

where $l_1, l_2, l_3, l_4, l_5, w_1, w_2, w_3, w_4, w_5$ are parameters and:

$$\hat{q}_2 = -q_2 \tag{110}$$

Observe that for the expression (109) to be valid it is required that:

$$\hat{q}_2 > 0 \qquad |\delta T_4| t \sqrt{\frac{w_4}{2\hat{q}_2}} \tag{111}$$

Let:

$$s_2 = -q_3 T_4^* \qquad s_4 = -q_2 T_4^* \tag{112}$$

We next review the conditions outlined in Theorem 1.

Condition-(ii):

$$\begin{aligned} \frac{\partial \phi^T}{\partial x} f(x) &= 2(-p_1(\lambda_a + \theta_a v_1^*) + p_3 \lambda_b)\ \delta T_2^2 \\ &+ 2(p_1 \lambda_a - p_3(\lambda_a + \theta_a v_1^*) + p_2 \lambda_b - p_3(\lambda_b + \theta_b v_3^*))\ \delta T_2 \delta T_4 \\ &+ 2(-p_2(\lambda_b + \theta_b v_3^*) + p_3 \lambda_a)\ \delta T_4^2 \end{aligned} \tag{113}$$

$$\begin{aligned} h(x)^T Q h(x) - l(x)^T l(x) &= (q_1 v_1^2 - l_1^2 - l_3^2 - l_4^2) \delta T_2^2 \\ &+ (2 q_3 v_1^* v_3^* - 2 l_4 l_5) \delta T_2 \delta T_4 + (-\hat{q}_2 (v_3^*)^2 - l_2^2 - l_5^2) \delta T_4^2 \end{aligned} \tag{114}$$

Equating (113) and (114) and matching the coefficients of the terms with the same power of δT_2 and δT_4, the following set of equations are obtained:

$$\begin{aligned} 2(-p_1(\lambda_a + \theta_a v_1^*) + p_3 \lambda_b) &= q_1 - l_1^2 - l_3^2 - l_4^2 \\ p_1 \lambda_a - p_3(\lambda_a + \theta_a v_1^*) + p_2 \lambda_b - p_3(\lambda_b + \theta_b v_3^*) &= q_3 v_1^* v_3^* - l_4 l_5 \\ 2(-p_2(\lambda_b + \theta_b v_3^*) + p_3 \lambda_a) &= -\hat{q}_2 (v_3^*)^2 - l_2^2 - l_5^2 \end{aligned} \tag{115}$$

Condition-(iii):

$$\frac{1}{2} G(x)^T \frac{\partial \phi}{\partial x} = \begin{bmatrix} \theta_a p_1 \delta T_2 + \theta_a p_3 \delta T_4 \\ \theta_b p_3 (T_3^* - T_4^* - \delta T_4) \delta T_2 + \theta_b p_2 (T_3^* - T_4^* - \delta T_4) \delta T_4 \end{bmatrix} \tag{116}$$

We substitute the values of s_2 and s_4 in (112) to obtain the following:

$$(QJ(x) + S)^T h(x) = \begin{bmatrix} s_1 v_1^*\ \delta T_2 + s_3 v_3^*\ \delta T_4 \\ q_3 v_1^*\ \delta T_2 \delta T_4 + q_2 v_3^*\ \delta T_4^2 \end{bmatrix} \tag{117}$$

$$-W(x)^T l(x) = \begin{bmatrix} -w_1 l_1 \delta T_2 - w_2 l_2 \delta T_4 \\ -\sqrt{\hat{q}_2} l_1\ \delta T_2 \delta T_4 - w_3 l_2 \delta T_4 - w_5 l_3 \delta T_2 \end{bmatrix} \tag{118}$$

Next, we match the coefficients of the terms with the same power of δT_2 and δT_4, to obtain the following set of equations:

$$\begin{aligned}\theta_a p_1 &= s_1 v_1^* - w_1 l_1 \\ \theta_a p_3 &= s_3 v_3^* - w_2 l_2 \\ \theta_b p_3 (T_3^* - T_4^*) &= -w_5 l_3 \\ \theta_b p_2 (T_3^* - T_4^*) &= -w_3 l_2 \\ -\theta_b p_3 &= q_3 v_1^* - \sqrt{\hat{q}_2}\, l_1 \\ \theta_b p_2 &= \hat{q}_2 v_3^*\end{aligned} \tag{119}$$

Condition-(iv):
Replacing s_4 in (112), we obtain:

$$R + J(x)^T S + S^T J(x) + J(x)^T Q J(x) = \begin{bmatrix} r_1 & r_3 + s_3(T_4^* + \delta T_4) \\ r_3 + s_3(T_4^* + \delta T_4) & r_2 + \hat{q}_2 (T_4^*)^2 - \hat{q}_2 \delta T_4^2 \end{bmatrix} \tag{120}$$

Also:

$$W(x)^T W(x) = \begin{bmatrix} w_1^2 + w_2^2 & w_2 w_3 + w_1 \sqrt{\hat{q}_2}\, \delta T_4 \\ w_2 w_3 + w_1 \sqrt{\hat{q}_2}\, \delta T_4 & w_4 + w_3^2 + w_5^2 - \hat{q}_2 \delta T_4^2 \end{bmatrix} \tag{121}$$

Equating (120) and (121) and matching the coefficients of the terms with the same power of δT_2, the following set of equations are obtained:

$$\begin{aligned} r_1 &= w_1^2 + w_2^2 \\ r_3 + s_3 T_4^* &= w_2 w_3 \\ s_3 &= w_1 \sqrt{\hat{q}_2} \\ r_2 + \hat{q}_2 (T_4^*)^2 &= w_4 + w_3^2 + w_5^2 \end{aligned} \tag{122}$$

We can now combine the set of Eqs. 115, 119, and 122 to determine the unknown parameters contained in the matrices Q, S, R, $l(x)$ and $W(x)$. The total amount of parameters is 23, while the number of equations [including (112)] is 15.

Proposition A.4 *(Dissipativity of heat exchanger, linear case). Consider heat exchanger 2 as depicted in Figure 5 with state space linear model. The heat exchanger is (Q, S, R)-dissipative with:*

$$\begin{aligned} Q &= \begin{bmatrix} q_1 & q_3 \\ q_3 & q_2 \end{bmatrix} t0 \\ S &= \begin{bmatrix} s_1 & s_2 \\ s_3 & s_4 \end{bmatrix} \\ R &= \begin{bmatrix} r_1 & r_3 \\ r_3 & r_2 \end{bmatrix} \end{aligned} \tag{123}$$

Proof: In the case of linear system, the conditions in Theorem 1 is simplified into:

$$\begin{aligned} &(i) \quad \phi(x) \geq 0,\ \phi(0) = 0 \\ &(ii) \quad \frac{\partial \phi^T}{\partial x} Ax = x^T C^T Q C x - x^T L^T L x \\ &(iii) \quad \frac{1}{2} B^T \frac{\partial \phi}{\partial x} = (QD + S)^T C x - W^T L x \\ &(iv) \quad R + D^T S + S^T D + D^T Q D = W^T W \end{aligned} \tag{124}$$

Condition-(i):
Let the storage function $\phi(x) :\ \mathrm{R}^n \to \mathrm{R}^+$ be:

$$\begin{aligned} \phi(x) &= x^T P x \\ \phi(x) &= [\,\delta T_2 \quad \delta T_4\,] \begin{bmatrix} p_1 & p_3 \\ p_3 & p_2 \end{bmatrix} \begin{bmatrix} \delta T_2 \\ \delta T_4 \end{bmatrix} \end{aligned} \tag{125}$$

Also let:

$$\begin{aligned} L &= \begin{bmatrix} l_1 & l_2 \\ l_3 & l_4 \end{bmatrix} \\ W^T &= \begin{bmatrix} w_1 & w_2 \\ w_3 & w_4 \end{bmatrix} \end{aligned} \tag{126}$$

where $l_1, l_2, l_3, l_4, w_1, w_2, w_3, w_4$ are parameters and:
Condition-(ii):

$$x^T (A^T P + PA) x = x^T C^T Q C x - x^T L^T L x \tag{127}$$

Equation 127 can be solved by evaluating:

$$A^T P + PA = C^T Q C - L^T L \tag{128}$$

Condition-(iii):

$$B^T P x = (QD + S)^T C x - W^T L x \tag{129}$$

Similarly, (129) can be solved by evaluating:

$$B^T P = (QD + S)^T C - W^T L \tag{130}$$

Condition-(iv):
This condition can be easily satisfied from the following equality:

$$R + D^T S + S^T D + D^T Q D = W^T W \tag{131}$$

We can now combine the set of matrix equalities (128), (130) and (131) to determine the unknown parameters contained in the matrices P, Q, S, R, L, and W. The total amount of matrices is 6, whereas the number of equalities is 3. ■

Numerical values for case study

Heat exchanger 1		Heat exchanger 3	
$A = 19.7$ m^2	$U = 600$ W m^{-2} K^{-1}	$A = 19.7$ m^2	$U = 600$ W m^{-2} K^{-1}
Process stream 1 (hot)		Process stream 1 (cold)	
$\rho = 750$ kg m^{-3}	$C_p = 2840$ J kg^{-1} K^{-1}	$\rho = 1000$ kg m^{-3}	$C_p = 4200$ J kg^{-1} K^{-1}
$T_{in} = 368$ K	$T_{out} = 343$ K	$T_{in} = 283$ K	$T_{out} = 302$ K
$\dot{m} = 6.25$ kg s^{-1}	$V = 0.245$ m^3	$\dot{m} = 5.63$ kg s^{-1}	$V = 0.202$ m^3
$\varepsilon = 0.8$	$T_{mix} = 348$ K	$\varepsilon = 0.8$	$T_{mix} = 298$ K
Process stream 2 (cold)		Process stream 2 (hot)	
$\rho = 1000$ kg m^{-3}	$C_p = 4200$ J kg^{-1} K^{-1}	$\rho = 750$ kg m^{-3}	$C_p = 2840$ J kg^{-1} K^{-1}
$T_{in} = 298$ K	$T_{out} = 313$ K	$T_{in} = 333$ K	$T_{out} = 313$ K
$\dot{m} = 5.63$ kg s^{-1}	$V = 0.0631$ m^3	$\dot{m} = 6.25$ kg s^{-1}	$V = 0.0630$ m^3
(Q, S, R)-dissipativity		(Q, S, R)-dissipativity	
$P = \begin{bmatrix} 16.3 & 3.51 \\ 3.51 & 5.52 \end{bmatrix}$	$Q = \begin{bmatrix} -10000 & 0 \\ 0 & -10000 \end{bmatrix}$	$P = \begin{bmatrix} 22.8 & 15.6 \\ 15.6 & 34.2 \end{bmatrix}$	$Q = \begin{bmatrix} -25000 & 0 \\ 0 & -5000 \end{bmatrix}$
$S = \begin{bmatrix} -2083 & -9000 \\ 0 & 13214 \end{bmatrix}$	$R = \begin{bmatrix} -56.7 & 31.7 \\ 31.7 & 23976 \end{bmatrix}$	$S = \begin{bmatrix} 2641 & -3447 \\ 0 & 5225 \end{bmatrix}$	$R = \begin{bmatrix} -5.88 & 0.661 \\ 0.661 & 1707 \end{bmatrix}$
Heat exchanger 2		Heat exchanger 4	
$A = 22.2$ m^2	$U = 600$ W m^{-2} K^{-1}	$A = 53.3$ m^2	$U = 500$ W m^{-2} K^{-1}
Process stream 1 (hot)		Process stream 1 (hot)	
$\rho = 750$ kg m^{-3}	$C_p = 2840$ J kg^{-1} K^{-1}	$\rho = 750$ kg m^{-3}	$C_p = 2840$ J kg^{-1} K^{-1}
$T_{in} = 348$ K	$T_{out} = 333$ K	$T_{in} = 313$ K	$T_{out} = 298$ K
$\dot{m} = 6.25$ kg s^{-1}	$V = 0.316$ m^3	$\dot{m} = 6.25$ kg s^{-1}	$V = 0.564$ m^3
Process stream 2 (cold)		Process stream 2 (cold)	
$\rho = 1000$ kg m^{-3}	$C_p = 4200$ J kg^{-1} K^{-1}	$\rho = 1000$ kg m^{-3}	$C_p = 4200$ J kg^{-1} K^{-1}
$T_{in} = 298$ K	$T_{out} = 313$ K	$T_{in} = 278$ K	$T_{out} = 288$ K
$\dot{m} = 4.23$ kg s^{-1}	$V = 0.0719$ m^3	$\dot{m} = 6.34$ kg s^{-1}	$V = 0.171$ m^3
(Q, S, R)-dissipativity		(Q, S, R)-dissipativity	
$P = \begin{bmatrix} 10.4 & 1.89 \\ 1.89 & 1.72 \end{bmatrix}$	$Q = \begin{bmatrix} -10000 & 0 \\ 0 & -10000 \end{bmatrix}$	$P = \begin{bmatrix} 34.0 & 11.0 \\ 11.0 & 10.8 \end{bmatrix}$	$Q = \begin{bmatrix} -10000 & 1 \\ 1 & -10000 \end{bmatrix}$
$S = \begin{bmatrix} 3.95 \times 10^3 & 3.16 \times 10^3 \\ 1.41 \times 10^3 & 5.65 \times 10^3 \end{bmatrix}$	$R = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	$S = \begin{bmatrix} 7.23 \times 10^3 & -1.03 \times 10^4 \\ -13.9 & 1.03 \times 10^8 \end{bmatrix}$	$R = \begin{bmatrix} 555 & 1.25 \times 10^5 \\ 1.25 \times 10^5 & -1.06 \times 10^{12} \end{bmatrix}$

HEN operability study

Overall open-loop (Q_{op}, S_{op}, R_{op})-dissipativity of HEN

$$Q_{op} = \begin{bmatrix} -10000 & 0 & 3950 & 1420 & -9000 & 0 & 0 & 0 \\ 0 & -10000 & 0 & 0 & 13200 & 0 & 0 & 0 \\ 3950 & 0 & -8290 & 0 & -3450 & 5230 & 0 & 0 \\ 1420 & 0 & 0 & -10000 & 0 & 0 & 0 & 0 \\ -9000 & 13200 & -3450 & 0 & -1020 & 0 & 0 & 0 \\ 0 & 0 & 5230 & 0 & 0 & -4450 & 7230 & -14 \\ 0 & 0 & 0 & 0 & 0 & 7230 & -10000 & 1 \\ 0 & 0 & 0 & 0 & 0 & -14 & 1 & -10000 \end{bmatrix} \quad (132)$$

$$S_{op} = \begin{bmatrix} -2.08 \times 10^3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0.661 & 0 & 3.16 \times 10^3 \\ 0 & 0 & 0 & 5.65 \times 10^3 \\ 31.7 & 2.64 \times 10^3 & 0 & 0 \\ 0 & 0 & 1.25 \times 10^5 & 0 \\ 0 & 0 & -1.03 \times 10^4 & 0 \\ 0 & 0 & 1.03 \times 10^8 & 0 \end{bmatrix} \quad (133)$$

$$R_{op} = \begin{bmatrix} -56.7 & 0 & 0 & 0 \\ 0 & -5.88 & 0 & 0 \\ 0 & 0 & -1.06 \times 10^{12} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (134)$$

Overall closed-loop (Q_{cl}, S_{cl}, R_{cl})-dissipativity of the HEN:

$$Q_{cl} = \begin{bmatrix} Q_{cl}^{11} & Q_{cl}^{12} \\ Q_{cl}^{21} & Q_{cl}^{22} \end{bmatrix},$$

where

$$Q_{\mathrm{cl}}^{11} = \begin{bmatrix} -6.44\times10^{12} & 0 & 3950 & 1420 & -9000 & 0 & 0 & 0 \\ 0 & -10000 & 0 & 0 & 13200 & 0 & 0 & 0 \\ 3950 & 0 & -8290 & 0 & -3450 & 5230 & 0 & 0 \\ 1420 & 0 & 0 & -10000 & 0 & 0 & 0 & 0 \\ -9000 & 13200 & -3450 & 0 & -1.41\times10^{13} & 0 & 0 & 0 \\ 0 & 0 & 5230 & 0 & 0 & -4445 & 7230 & -14 \\ 0 & 0 & 0 & 0 & 0 & 7230 & -6.44\times10^{12} & 1 \\ 0 & 0 & 0 & 0 & 0 & -14 & 1 & -10000 \end{bmatrix}$$

$$Q_{\mathrm{cl}}^{12} = \begin{bmatrix} -0.00440 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0.661 & 0 \\ 0 & 0 & 0 \\ 31.7 & 373 & 0 \\ 0 & 0 & 1.25\times10^{5} \\ 0 & 0 & 1.75\times10^{7} \\ 0 & 0 & 1.03\times10^{8} \end{bmatrix} \tag{135}$$

$$Q_{\mathrm{cl}}^{21} = Q_{cl}^{12T}$$

$$Q_{\mathrm{cl}}^{22} = \begin{bmatrix} -56.7 & 0 & 0 \\ 0 & -5.88 & 0 \\ 0 & 0 & -1.06\times10^{12} \end{bmatrix}$$

$$S_{\mathrm{cl}}^{T} = [0 \quad 0 \quad 3.16\times10^{3} \quad 5.65\times10^{3} \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0] \tag{136}$$

$$R_{\mathrm{cl}} = 0 \tag{137}$$

Manuscript received Feb. 13, 2008, and revision received Sept. 26, 2008.